Executing Task Graphs Using Work-Stealing

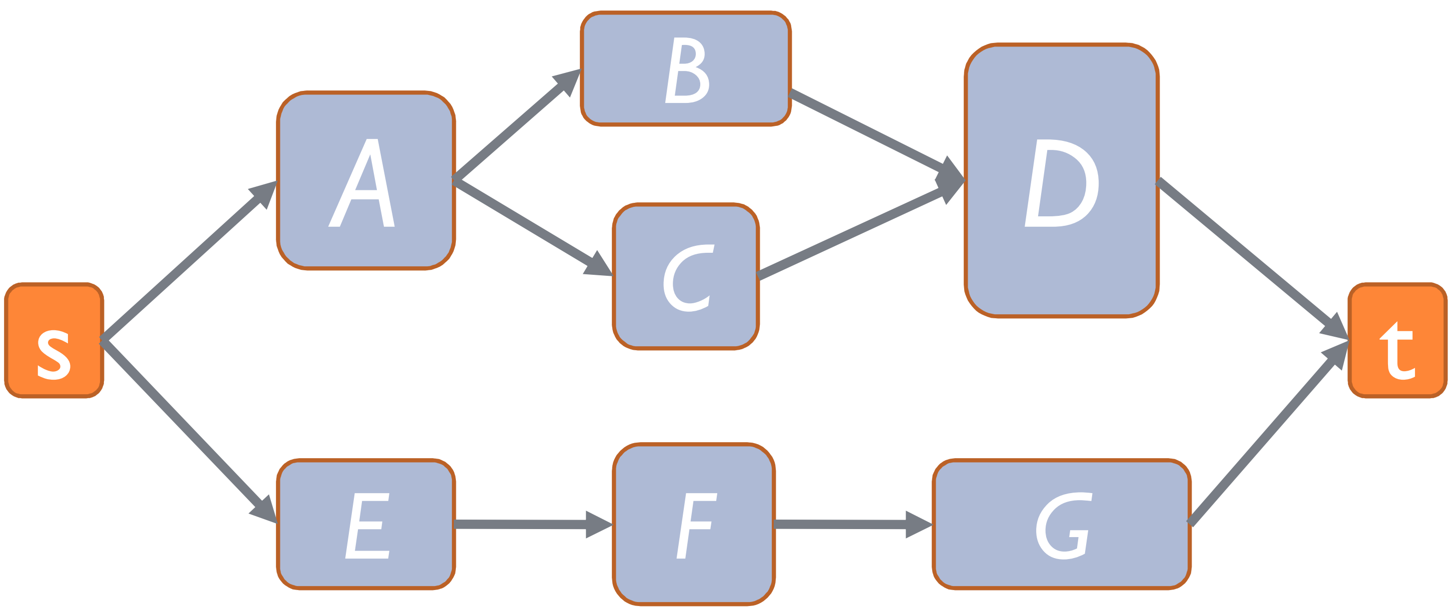
Jim Sukha
MIT CSAIL
IPDPS, 4/21/2010

Kunal Agrawal (Washington University in St. Louis)
Charles E. Leiserson (MIT)

Task Graphs

A *task graph* is a directed acyclic graph (dag) where

- Every node *A* is a task requiring computation,
- Every edge (*A*, *B*) means that the computation of *B* depends on the result of *A*'s computation.

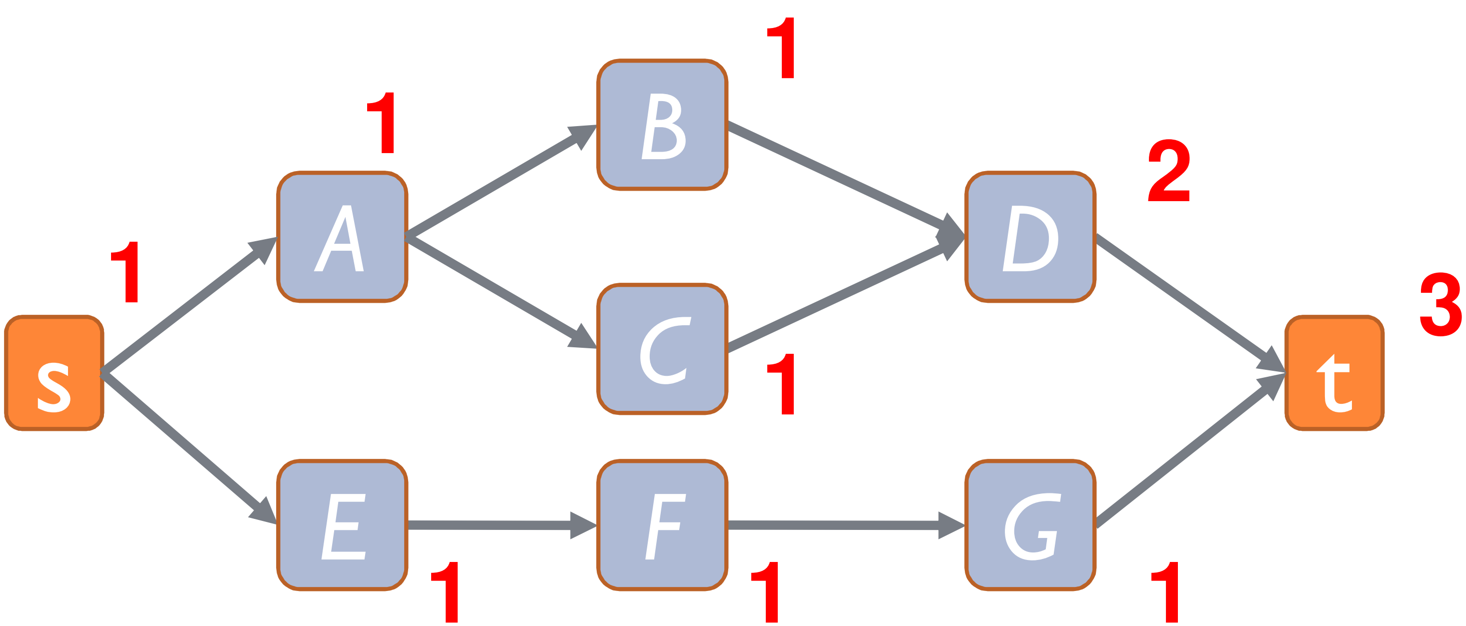


Example: Counting Paths

Counting the number of paths through a dag can be expressed as a task graph.

Source starts with value 1.

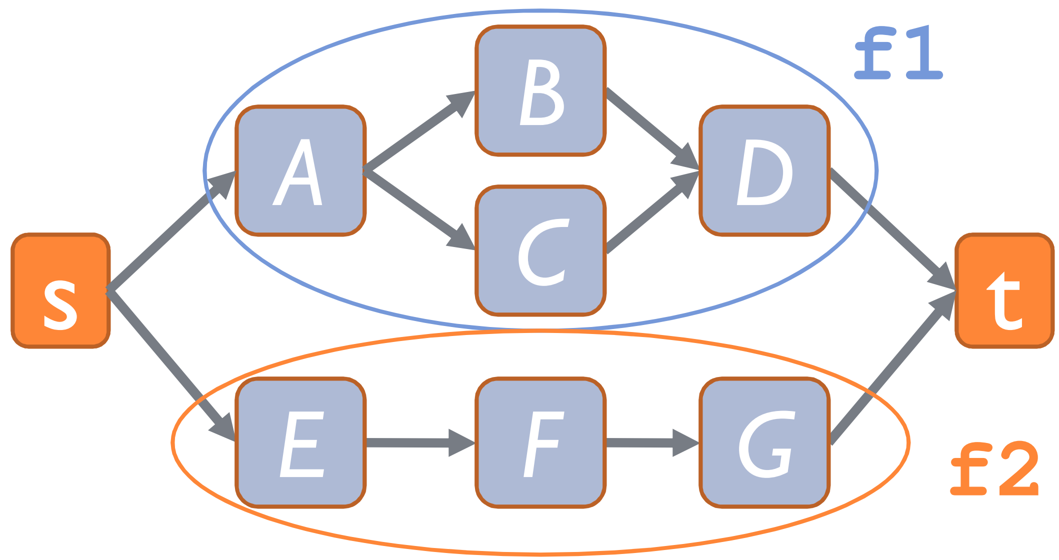
The value of a node *X* is the sum of the values of *X*'s immediate predecessors.



Fork-Join Languages

Some task graphs can be executed in parallel using a fork-join language such as Cilk++.

```
void f1() {
   A();
   cilk_spawn B();
   C();
   cilk_sync;
   D();
}
```



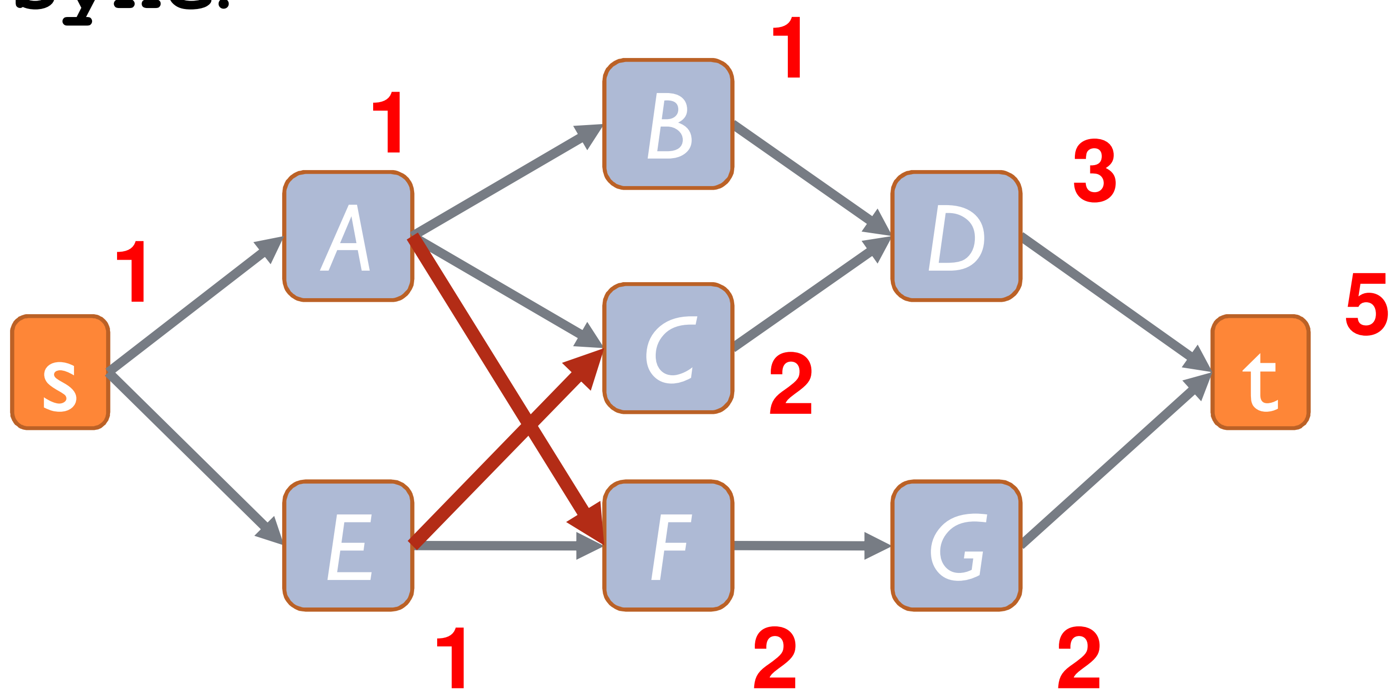

```
void f2() {
   E(); F(); G();
}
```

```
int main() {
   cilk_spawn f1();
   f2();
   cilk_sync;
}
```

Graphs with Arbitrary Dependencies

Unfortunately, one can not directly express task graphs with **arbitrary** dependencies using only **`spawn`** and **`sync`**.

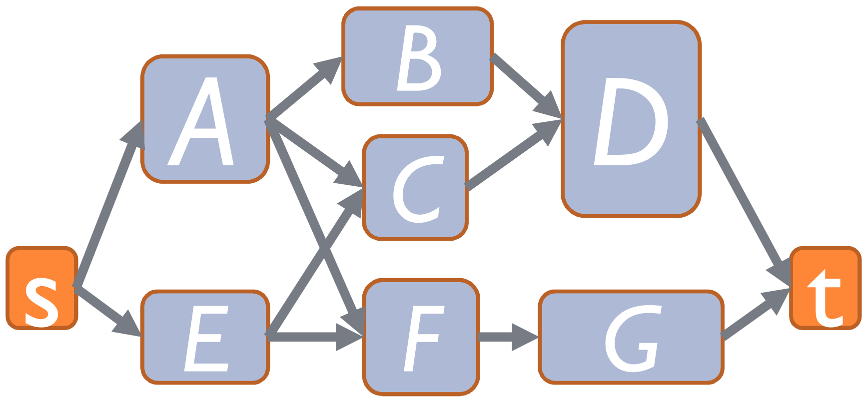
Counting paths in a non-series-parallel dag.



Question: Can we efficiently execute arbitrary task graphs in parallel in a fork-join language such as Cilk++?

Our Contributions: Nabbit

We are developing Nabbit, a Cilk++ library for executing task graphs with arbitrary dependencies.



- Nabbit is built on top of Cilk++. It utilizes Cilk++'s provably-efficient work-stealing scheduler without any modification to the Cilk++ runtime.
- Using Nabbit, the computation of an individual task graph node can itself be parallel.

Provable Bounds for Nabbit

We are developing Nabbit, a Cilk++ library for executing task graphs with arbitrary dependencies.

- Nabbit offers provable bounds on the time required for parallel execution of (static and dynamic) task graphs.
- The time bounds for Nabbit are asymptotically optimal for task graphs whose nodes have constant in-degree and out-degree.

Outline

- Static Task Graphs using Nabbit
- Nabbit Implementation
- Completion Time Bound
- Dynamic Task Graphs and Other Extensions

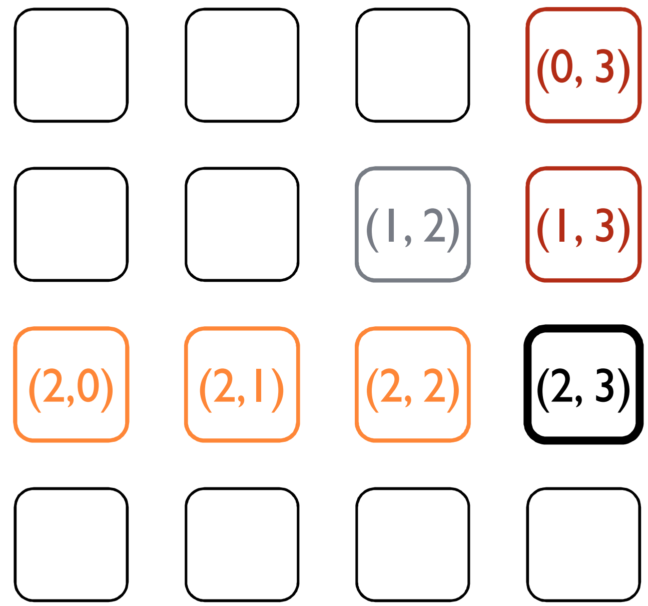
Dynamic-Programming Example

Generic dynamic programs can often be expressed as a task graph.

$$M(i, j) = \max \begin{cases} g_1(M(i-1, j-1)) \\ E(i, j) \\ F(i, j) \end{cases}$$

$$E(i, j) = g_2(\, M(1, j),\ M(2, j), \ldots, M(i-1, j)\,)$$

$$F(i, j) = g_3(\, M(i, 1),\ M(i, 2), \ldots, M(i, j-1)\,)$$



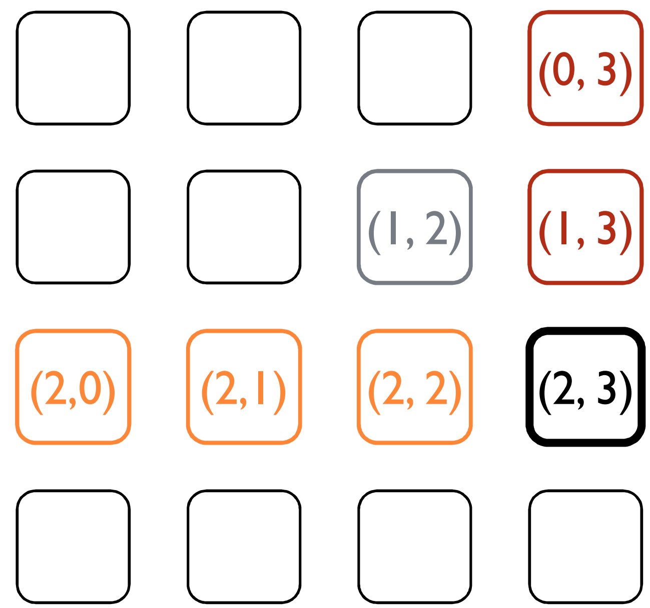
Static Task Graphs

For this example, we can use a ***static task graph***, i.e., a task graph where the structure of the dag is known before the execution begins.

$$M(i, j) = \max \begin{cases} g_1(M(i-1, j-1)) \\ E(i, j) \\ F(i, j) \end{cases}$$

$$E(i, j) = g_2(\ M(1, j),\ \ M(2, j), \ldots,\ M(i-1, j)\ )$$

$$F(i, j) = g_3(\ M(i, 1),\ \ M(i, 2), \ldots,\ M(i, j-1)\ )$$



Create a node for every cell $M(i, j)$.

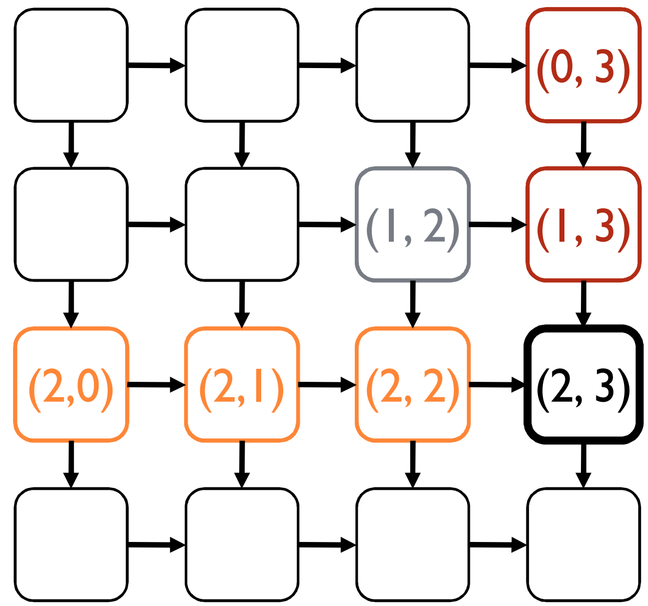
Static Task Graphs

For this example, we can use a ***static task graph***, i.e., a task graph where the structure of the dag is known before the execution begins.*

$$M(i, j) = \max \begin{cases} g_1(M(i-1, j-1)) \\ E(i, j) \\ F(i, j) \end{cases}$$

$$E(i, j) = g_2(\, M(1, j),\ M(2, j), \ldots, M(i-1, j)\,)$$

$$F(i, j) = g_3(\, M(i, 1),\ M(i, 2), \ldots, M(i, j-1)\,)$$



Create a node for every cell $M(i, j)$. Then add dependency edges.

* In Nabbit, static task graphs still require ***dynamic scheduling***.
We assume the compute time for each node may be unknown.

Interface for Static Nabbit

For static task graphs, each task graph node is derived from Nabbit's **DAGNode** class, and overrides the node's **Compute()** method.

```
class Mnode: public DAGNode {
  Mnode(int key, MDag* dag);
  void Compute();
};
```

Typically, each node needs to know its identity, and global parameters for task graph.

```
class MDag {
  int N; int *M;
  Mnode* g;
  MDag(int N_, int* M_);
};
```

Programmer builds their own task graph.

Constructing a Static Task Graph

Programmers use Nabbit's **`AddDep`** method to specify dependencies between task graph nodes.

```
class MDag {
  int N; int* s; Mnode* g;
  MDag(int n_, int* M_) : N(N_), M(M_) {
    g = new Mnode[N*N];

    for (int i = 0; i < N; i++){
      for (int j = 0; j < N; j++) {
        int k = N*i+j;
        g[k].key = k; g[k].dag = this;
        if (i > 0) g[k].AddDep(&Mnode[k-N]);
        if (j > 0) g[k].AddDep(&Mnode[k-1]);
      }
    }
  }
};
```

Allocate nodes

M(i, j) has edges from M(i-1, j) and M(i, j-1).

Implementing Task Nodes

Task graph nodes inherit from a **DAGNode** class, and override the node's **Compute()** method.

```
class Mnode: public DAGNode {
  int i, j;

  void Compute() {
    int z = INFINITY;
    int Eij = calcE(dag->M, i, j);
    int Fij = calcF(dag->M, i, j);
    if ((i > 0) && (j > 0))
      z = g1(M, i, j);
    dag->M[key] = min(z, Eij, Fij);
  }
};
```

One can call other Cilk functions inside the **Compute()** method, including methods that **spawn** and **sync**.

Outline

- Static Task Graphs using Nabbit
- Nabbit Implementation
- Completion Time Bound
- Dynamic Task Graphs and Other Extensions

Static Nabbit Implementation

Nabbit uses a simple algorithm to execute static task graphs in parallel. Each node

1. Maintains a count of the # of its immediate predecessors that are still incomplete. (Each node keeps a join counter.)
2. Notifies its immediate successors in parallel after it is computed.
3. Recursively computes any successors which become ready.

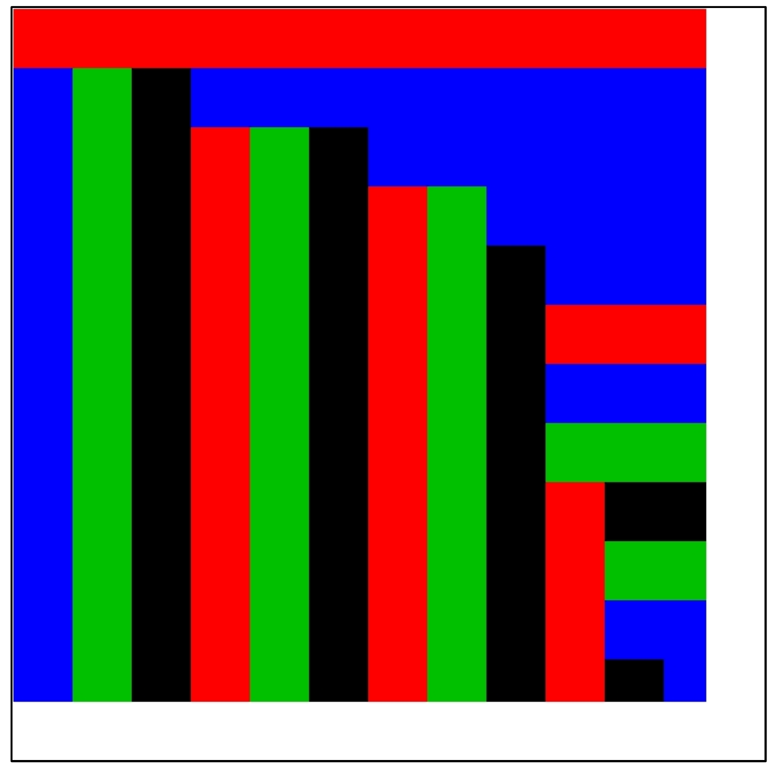
```
void ComputeAndNotify() {
  this->Compute();
  cilk_for (int q = 0;
            q < successors().size();
            q++) {
    DAGNode* Y = successors[q];
    int val = AtomicDecAndFetch(Y.join);
    if (val == 0) Y.ComputeAndNotify();
  }
}
```

Start execution by calling **ComputeAndNotify** from the source (root) node.

Work-Stealing in Nabbit

Nabbit is able to rely on Cilk++'s work-stealing scheduler to load-balance the computation.

- When a processor runs out of work, it tries to steal work from other processors.
- Nabbit spawns task nodes in a way that makes the Cilk++ runtime likely to steal nodes along the critical path of the task graph.

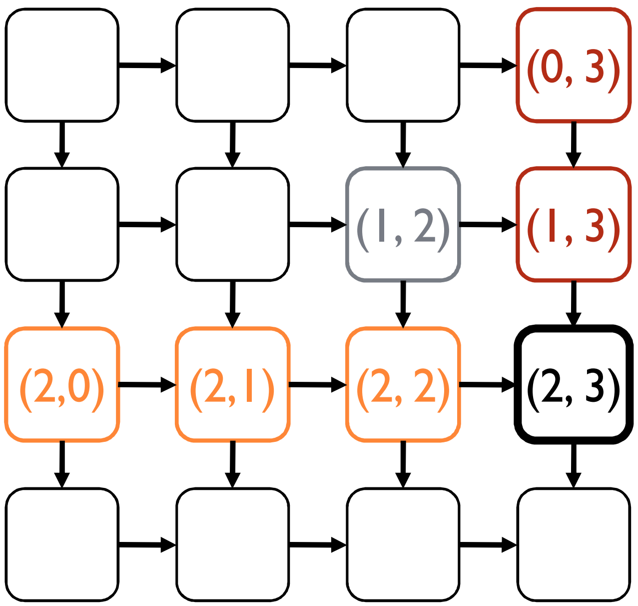


Sample Dynamic-Program Execution with *P*=4

Smith-Waterman Dynamic Program

As a benchmark, we consider a dynamic program modeling the Smith-Waterman algorithm with a generic penalty gap:

$$M(i, j) = \max \begin{cases} M(i-1, j-1) + s(i, j) \\ E(i, j) \\ F(i, j) \end{cases}$$



$$E(i, j) = \max_{k \in \{0, 1, \ldots i-1\}} M(k, j) + \gamma(i-k)$$

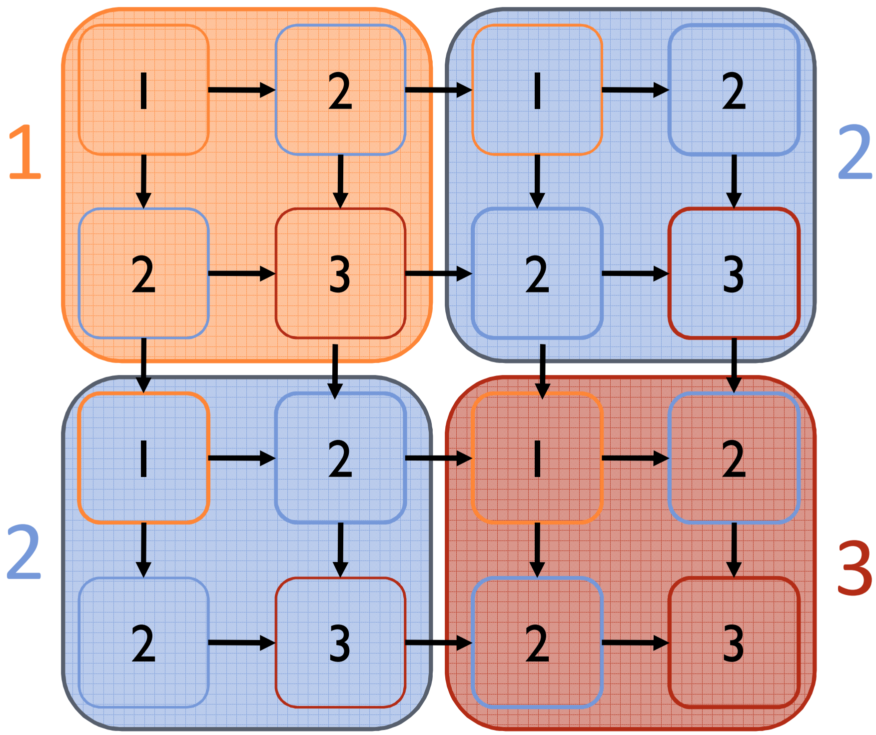
$$F(i, j) = \max_{k \in \{0, 1, \ldots j-1\}} M(i, k) + \gamma(j-k)$$

In this example, $s(i, j)$ and $\gamma(k)$ are constant arrays.

Comparison with Divide-and-Conquer

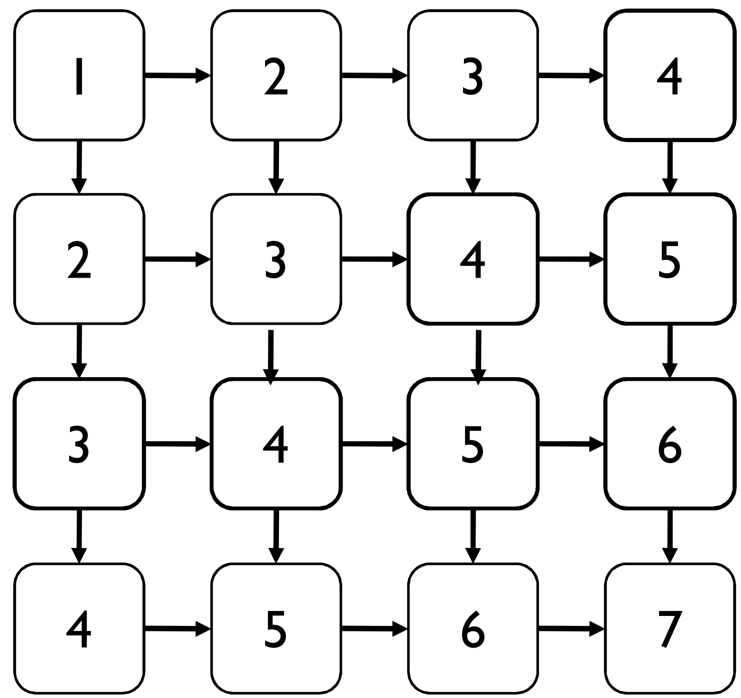
For the dynamic program, we compare the task graph evaluation using Nabbit with alternative algorithms.

K-way divide-and-conquer



K=2

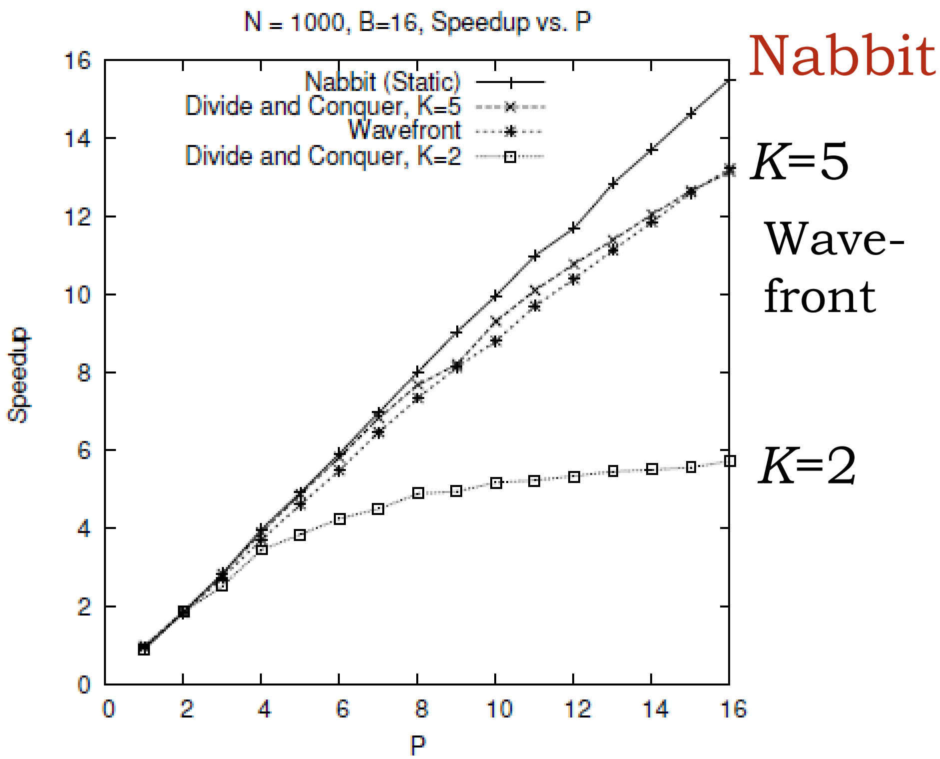
Wavefront (Synchronous)

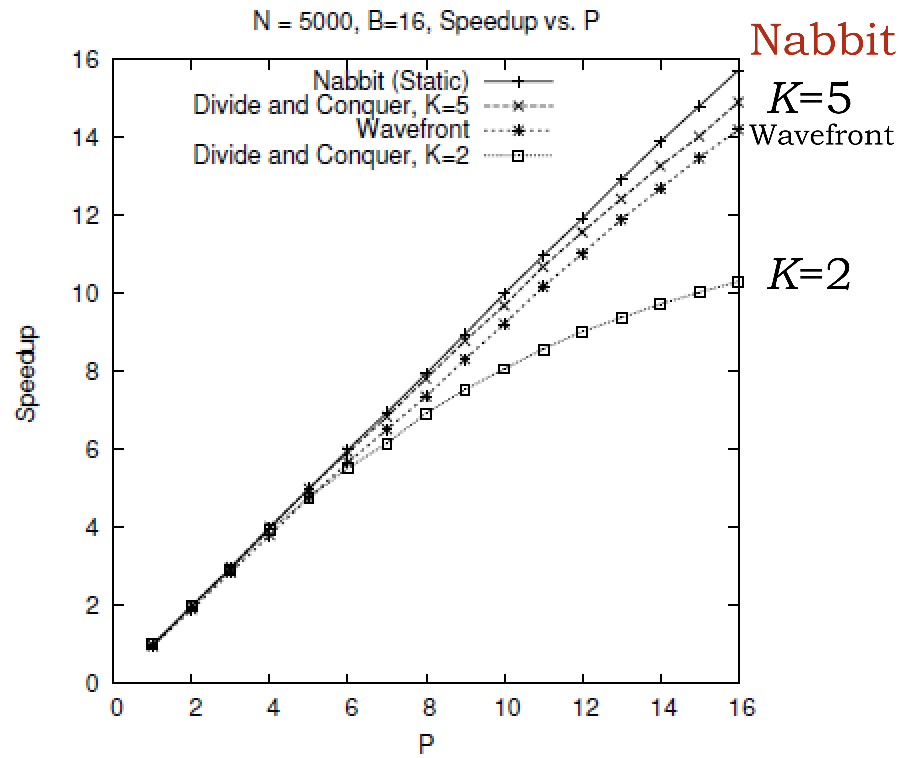


For all algorithms, base case is blocks of size *B* by *B*. Grid is arranged in a cache-oblivious layout.

16-core AMD Barcelona

Comparing implementations of the dynamic program





Serial Running Time = 4.4 s
c = 4.4e-9 if $T_S \sim= cN^3$

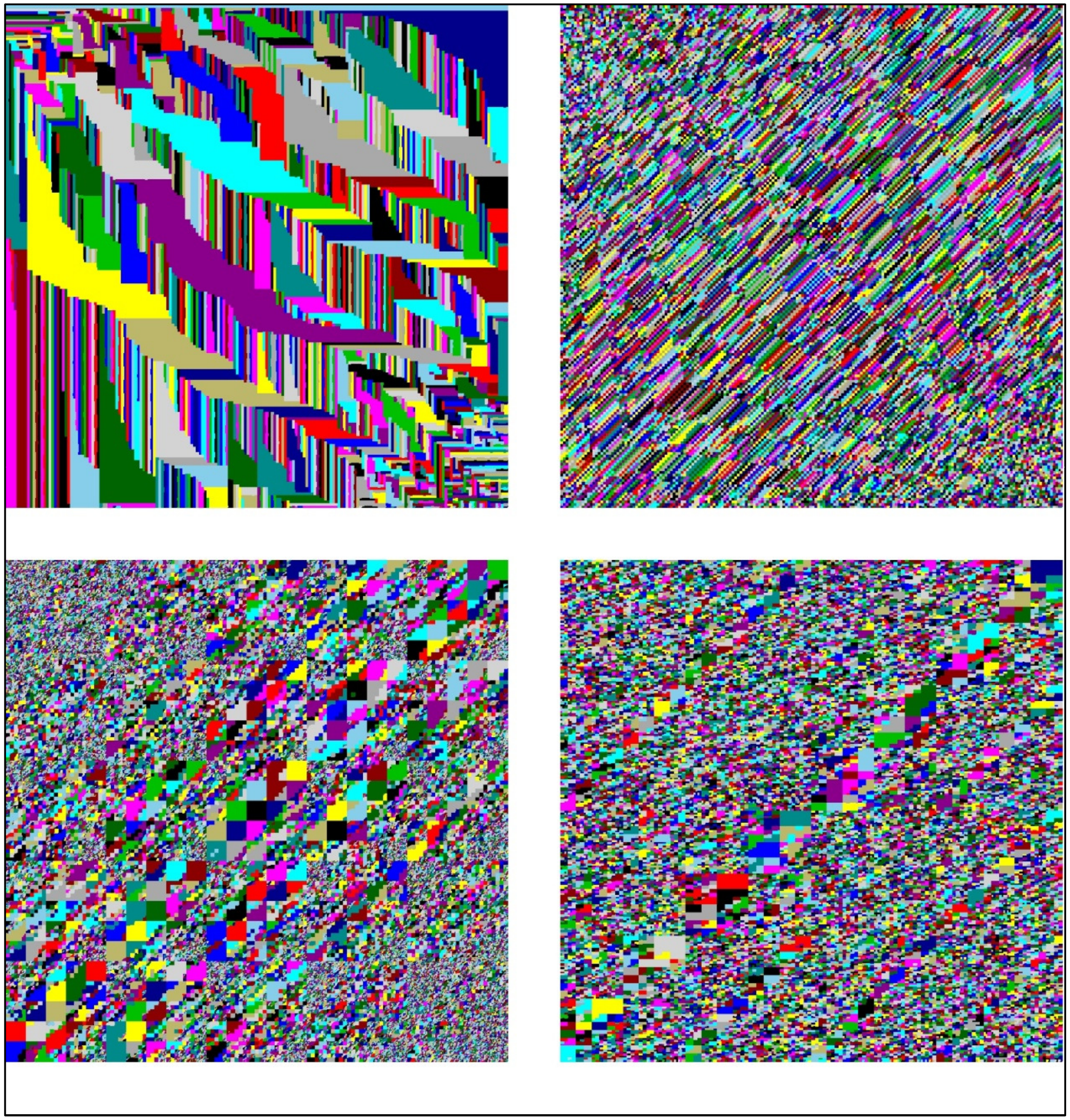
Serial Running Time = 664 s
c = 5.3e-9 if $T_S \sim= cN^3$

Opteron Processor 8354: 2.2 Ghz

Comparison, N=3000, P=16

Nabbit

Wavefront



K=5

K=2

Outline

- Static Task Graphs using Nabbit
- Nabbit Implementation
- Completion Time Bound
- Dynamic Task Graphs and Other Extensions

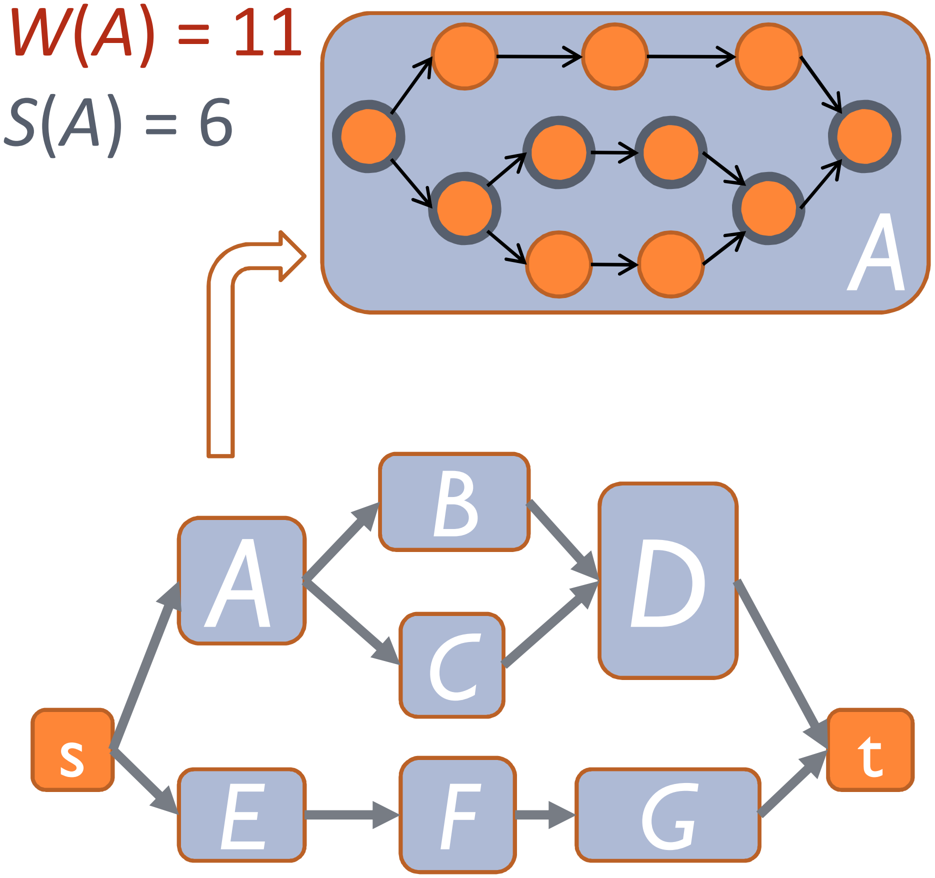
Definitions

Let $D=(V, E)$ be a task graph to execute.
Consider the execution dag associated with the **`Compute()`** method of a task node $A \in V$.

$W(A)$: the work of A
(# of nodes in execution dag)

$S(A)$: the span of A
(length of longest path in dag)

$W(A) = 11$
$S(A) = 6$



M: # of task nodes on longest path through task graph D.

Δ: maximum degree of any task node

$M = 5$
$\Delta = 2$

Work and Span of a Task Graph

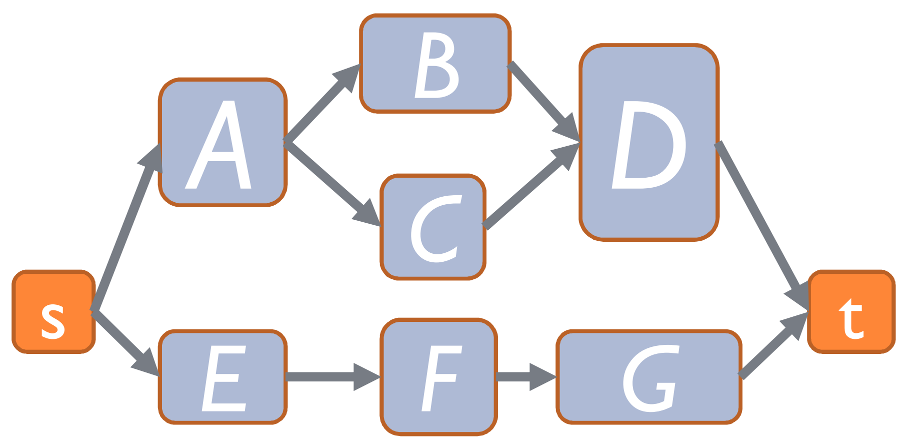
We can define a "total" work and span for the task graph execution. Define T_1 and T_∞ as:

$$T_1 = \sum_{A \in V} W(A) + O(E)$$

$$T_\infty = \max_{\substack{\text{all paths } p \\ \text{through } D}} \left\{ \sum_{A \in p} (S(A) + O(1)) \right\}$$

Any execution of the task graph on P processors requires time at least:

$$\max\{\, T_1/P,\ T_\infty \,\}.$$



Completion Time for Static Nabbit

THEOREM 1: Nabbit executes a static task graph $D = (V, E)$ on P processors in expected time

$$O\left(\frac{T_1}{P} + T_\infty + M \lg \Delta + C(D)\right)$$

where $C(D) = O\left(\left(\frac{E}{P} + M\right) \min\{\Delta, P\}\right)$.

$T_1/P + T_\infty$: Bound for ordinary Cilk-like work-stealing

$M \lg \Delta$: span of notifying task node successors

$C(D)$: worst-case contention for atomic decrements.

(min{Δ, P}: time a decrement can wait)

Theorem is asymptotically optimal when $\Delta = \Theta(1)$.

M: # of nodes on longest path through task graph D.

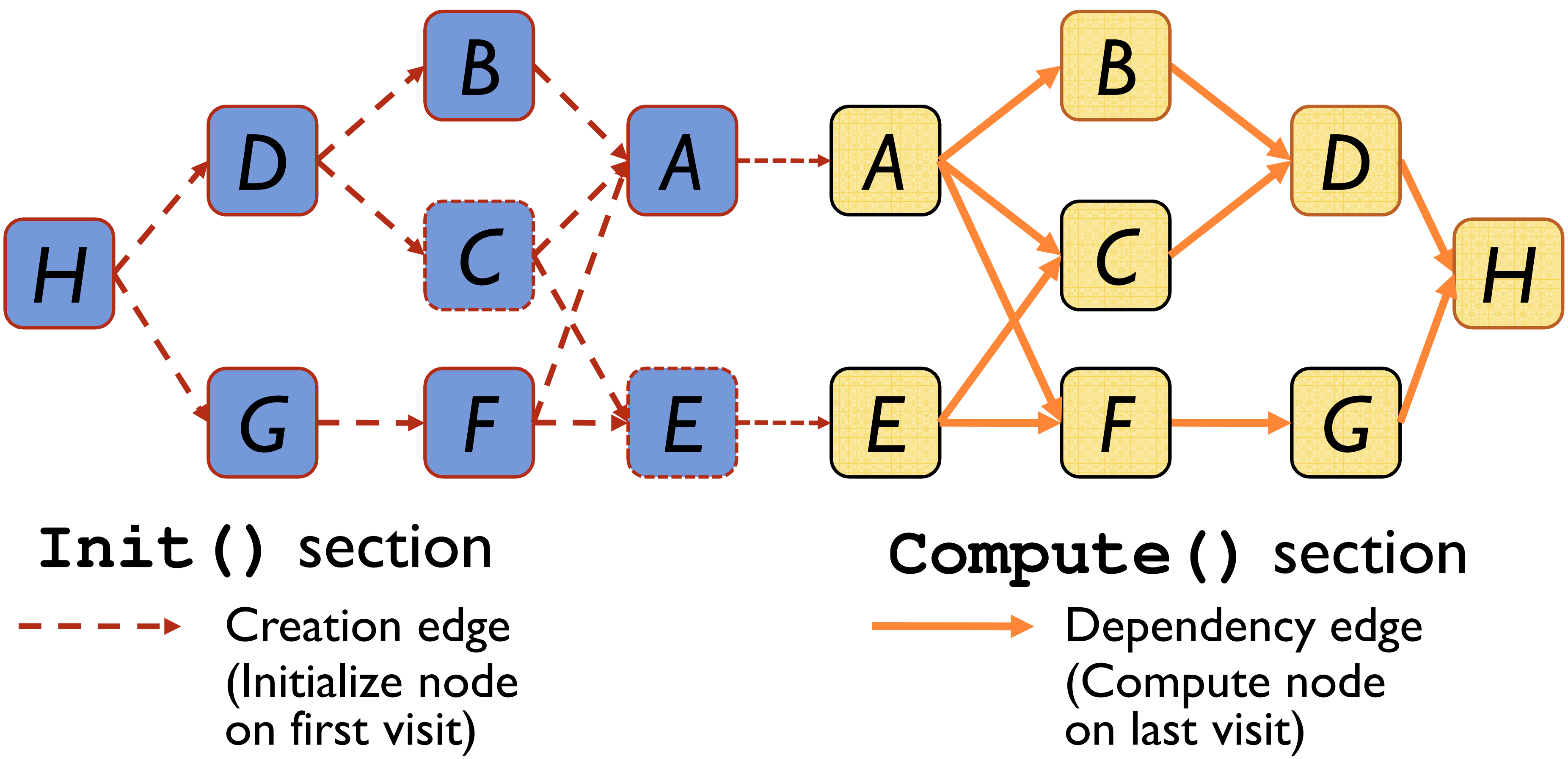
Δ: maximum degree of any task node

Outline

- Static Task Graphs using Nabbit
- Nabbit Implementation
- Completion Time Bound
- Dynamic Task Graphs and Other Extensions

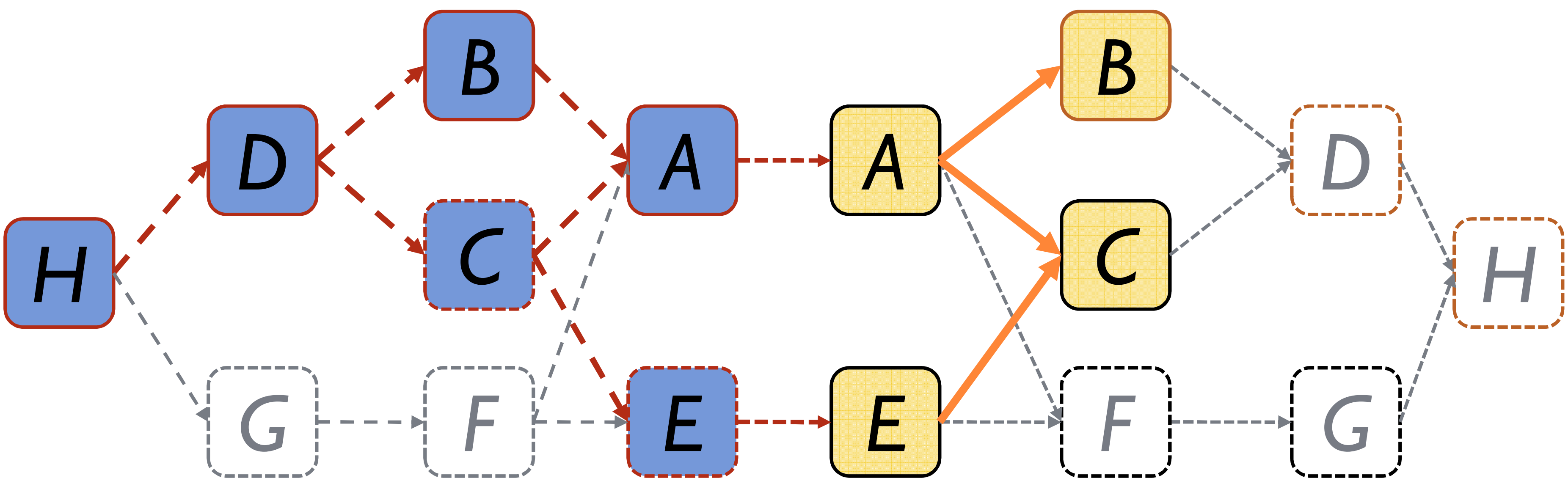
Dynamic Nabbit

Nabbit also supports *dynamic task graphs*. Roughly, a dynamic task graph can be thought of as performing a parallel traversal of a two-phase dag, where the first **`Init()`** phase creates new nodes.



Complications for Dynamic Nabbit

Dynamic task graphs are more complicated because `Init()` and `Compute()` happen concurrently.



`Init()` section

Creation edge
(Initialize node
on first visit)

`Compute()` section

Dependency edge
(Compute node
on last visit)

Completion Time for Dynamic Nabbit

THEOREM 2: Nabbit executes a dynamic task graph $D = (V, E)$ on P processors in expected time

$$O\left(\frac{T_1}{P} + T_\infty + M\Delta + C(D)\right)$$

where $$C(D) = O\left(\left(\frac{E}{P} + M\right)\min\{\Delta, P\}\right).$$

T_1 and T_∞ are modified to account for `Init()` for each node.

$M\Delta$: weaker bound because all edges in the graph are not known ahead of time.

Topics for Future Investigation

We are interested in possibly extending Nabbit in several directions:

- Strongly Dynamic Task Graphs
  - `Compute()` of a task node can generate a new task.
- Reusing Nodes and Garbage Collection
- Hierarchical Task Graphs
- Runtime/Compiler Support for Nabbit

Applications for Nabbit?

We are interested in possibly extending Nabbit in several directions:

- Strongly Dynamic Task Graphs
  - `Compute()` of a task node can generate a new task.
- Reusing Nodes and Garbage Collection
- Hierarchical Task Graphs
- Runtime/Compiler Support for Nabbit
- Applications!

The value of these possible extensions to Nabbit depends on programs that use static or dynamic task graphs.

We value any feedback regarding potential applications!

Questions?