Load Regulating Algorithm for Static-Priority Task Scheduling on Multiprocessors

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OUTLINE

- Introduction
- Problem Statement
- Task Model
- Scheduling
  - Feasibility Condition
  - Multiprocessor scheduling
- IBPS Scheduling
- Conclusion

Introduction

- ***Real-Time Systems*** have timing constraints
- Applications of real-time systems are often modeled as a collection of periodic tasks
- Timing constraints (e.g. deadlines) are stringent in ***hard*** real-time systems
- ***Scheduling*** can ensure that all deadlines are met

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Problem Statement

- *Given*
  - *a collection of tasks*
  - *a collection of available processors*
- *the **multiprocessor scheduling problem** is to determine*
  - *whether the tasks can be partitioned among the processors such that all deadlines are met*

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Task Model

- Application is modeled as ***a set of periodic tasks***.
  - A task set $\Gamma = \{\tau_1, \tau_2, \ldots, \tau_n\}$ is to be executed on ***m processors***

- Each task τ_i has
  - A ***period*** T_i (inter-arrival time)
  - A ***worst-case execution time*** C_i

- Each invocation requires C_i units of execution time before next period

Task Model (cont.)

- Rate-Monotonic (RM) pre-emptive scheduler is used in each processor

- Using RM scheduling, each task τ_i has a priority.
  - ***The shorter the period, the higher the priority.***

- ***Utilization*** of a task τ_i is $\mathbf{u_i = C_i/T_i}$

- The ***total utilization*** of a set Γ of tasks is $\mathbf{U(\Gamma) = \sum u_i}$

Example Task Set

τ_i	C_i	T_i
τ_1	5	10
τ_2	2	7
τ_3	6	15

$u_1 = 5/10 = 0.5$ $u_2 = 2/7 = 0.285$ $u_3 = 6/14 = 0.428$

Total utilization, $U(\Gamma) = u_1 + u_2 + u_3$

τ_2 has the ***highest*** priority and τ_3 has the ***lowest*** prirority.

Rephrased Problem Statement

- How can we guarantee that a set of tasks Γ is ***RM schedulable*** on ***m*** processors?

IBPS

Interval-**B**ased **P**artitioned **S**cheduling

The scheduling guarantee using IBPS is given using a ***feasibility condition.***

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Feasibility Condition

Feasibility Condition of a scheduling algorithm is used to determine (offline) whether all the tasks meet their deadlines during run-time.

- ***Necessary and Sufficient, or***
- ***Sufficient only***

Necessary and sufficient feasibility test is precise but has higher time complexity

Sufficient Feasibility Condition

Utilization based sufficient feasibility condition of algorithm *A* has the following form:

A = Uniprocessor Rate-Monotonic(RM) Scheduling (Liu and Layland, 1973)

if $U(\Gamma) \leq n(2^{1/n} - 1)$, then Γ is RM-schedulable on uniprocessor.

A = Multiprocessor Rate-Monotonic(RM) First-Fit Scheduling (D. Oh 1998)

if $U(\Gamma) \leq 0.41m$, then Γ is RM-schedulable on *m* processors.

IBPS: Sufficient Feasibility Condition

If $U(\Gamma) \leq 0.552m$, then then Γ is IBPS-schedulable on m processors.

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Multiprocessor Scheduling

- Two main approaches
  - — **Global** (no task assignment, global queue, migration)
  - — **Partitioned** (task assignment, local queue, no migration)
- Neither global nor partitioned scheduling can have achievable system utilization more than 50% for static-priority tasks (D. Oh et al. 1998, B. Andersson et al. 2001)

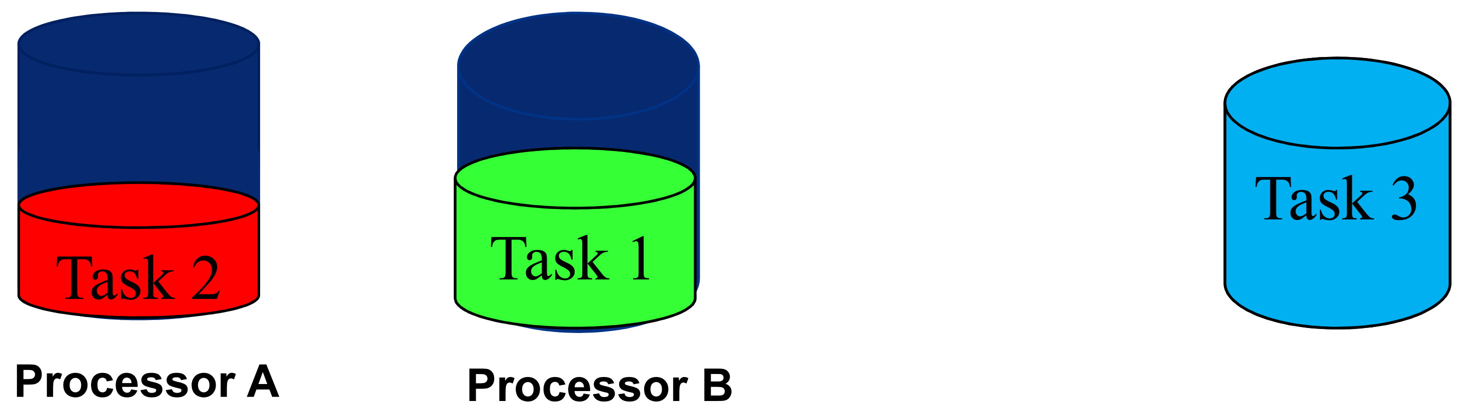
Task-Splitting Partitioned Method

- A *variation of partitioned scheduling* based on **task-splitting** approach can achieve more than 50%
  - When a task can not be assigned to a processor, it is split (i.e. migrated during runtime)
  - A bounded number of tasks are migrated

Traditional Partitioned Scheduling

We assume *Task 2*, *Task 1* and *Task 3* be the ordering of the tasks to assign to the processors A and B.

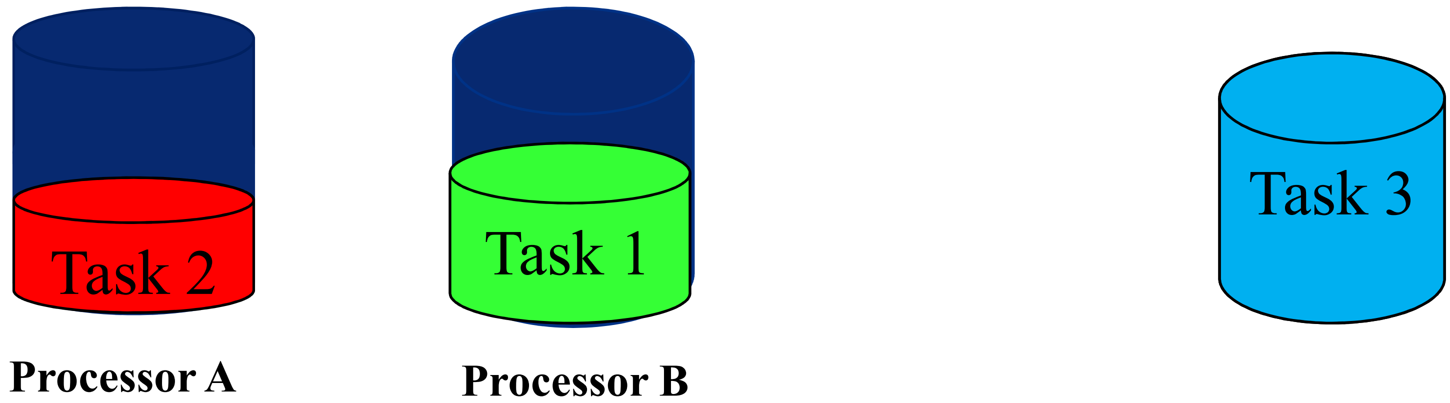
Traditional Partitioned Scheduling



Partition Fails!

Task 3 cannot be assigned to any processor because size of Task 3 is too large

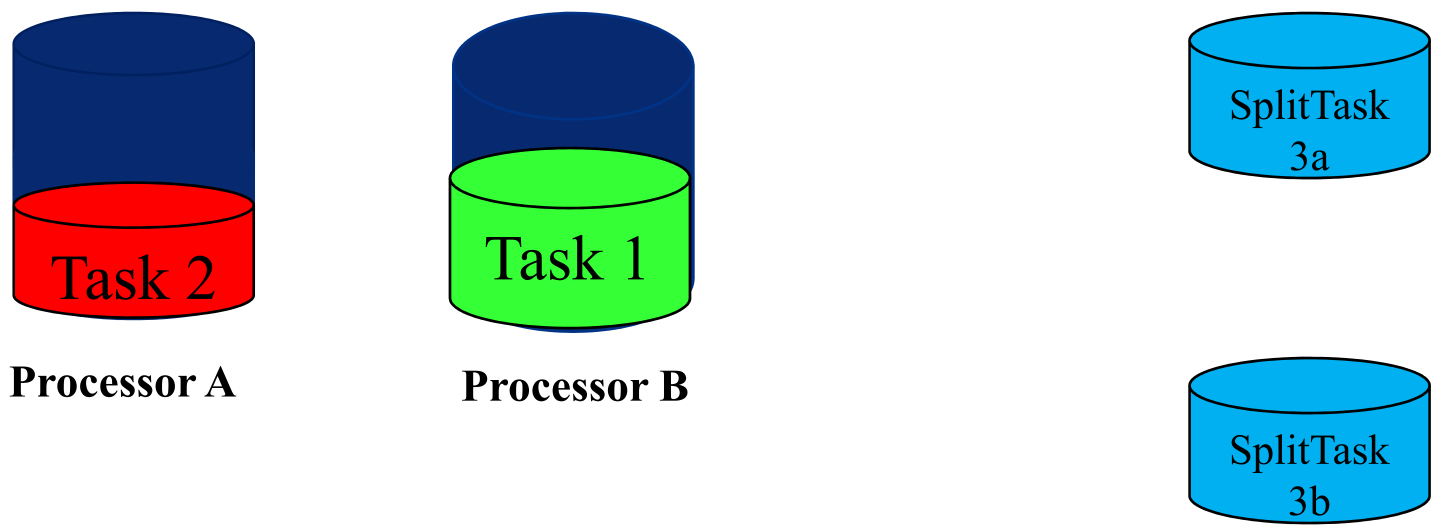
Task-Splitting Partitioned Scheduling



Different subtasks of Task 3 can be assigned to different processors.

To construct the subtasks, we split Task 3.

Task-Splitting Partitioned Scheduling



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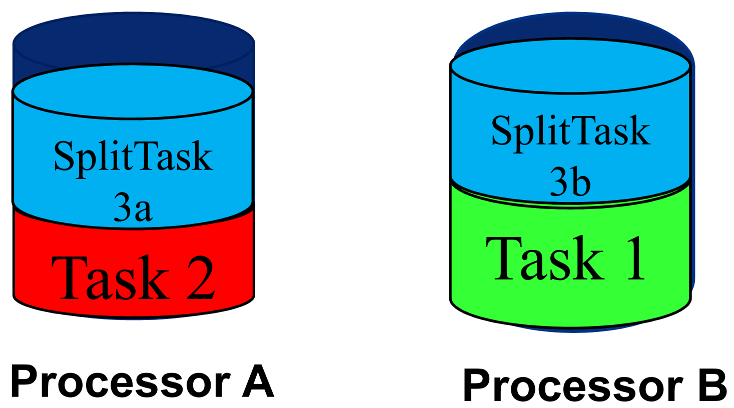
Task-Splitting Partitioned Scheduling



Partition Success!

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IBPS: Basic Idea

- ***n** tasks are grouped in seven utilization subintervals.*
- ***n*** tasks are assigned to ***m*** processor in ***three phases***
  - First two phases has load regulation
- Each processor executes tasks using RM scheduling

IBPS: Basic Idea

- The total utilization in ***each*** processor in the first two phases is greater than 55.2% (***load regulation***)

- All unassigned tasks are assigned in the third phase.

- A task that cannot be assigned to a processor is ***split***.
  - Split a task in exactly two parts, and
  - Each processor only has at most one split task (i.e. m/2 split tasks)

IBPS: Tasks Grouping in Subintervals

$\tau_1, \tau_2, \ldots \tau_n$

I_7 $I_7=(0, 0.136]$

I_6 $I_6=(0.136, 0.184]$

I_5 $I_5=(0.184, 0.221]$

I_4 $I_4=(0.221, 0.276]$

I_3 $I_3=(0.276, 0.368]$

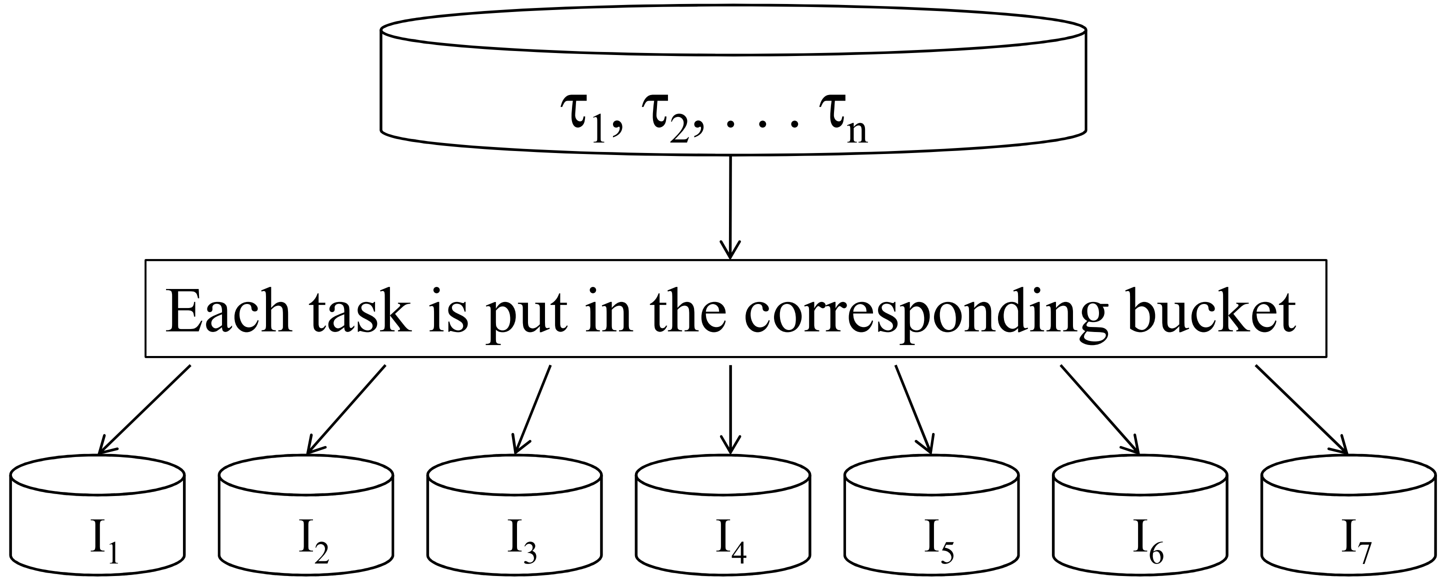
I_2 $I_2=(0.368, 0.552]$

I_1 $I_1=(0.552, 1.0]$

The utilization interval (0, 1.0] is divided into ***seven*** utilization subintervals

Each task utilization is within one of the seven utilization subintervals

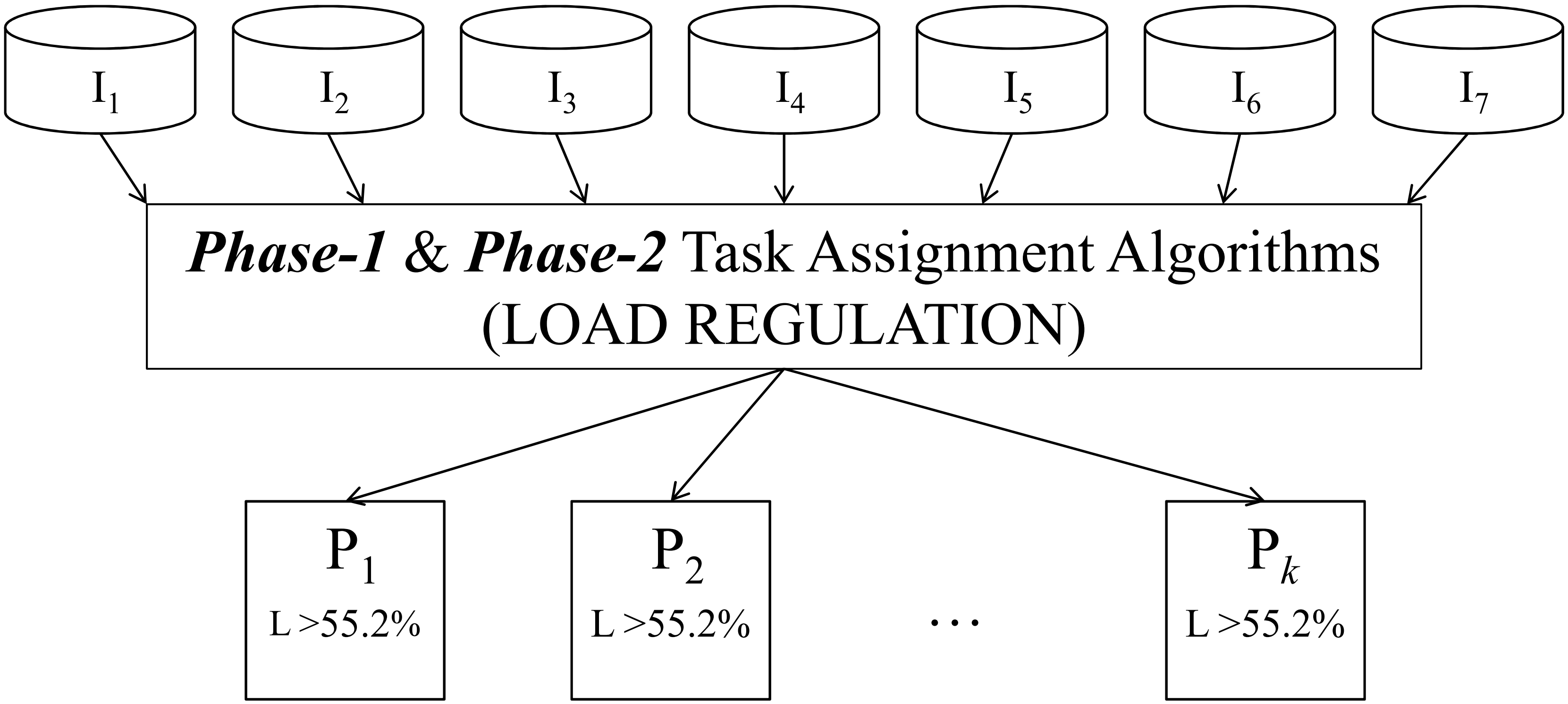
IBPS: Seven Utilization Subintervals



- Each subinterval has lower and upper bound
  - For example, I_2=(0.368, 0.552]

- If there are 3 tasks in I_2, then the min and max utilization are (3×0.368) and (3×0.552), respectively.

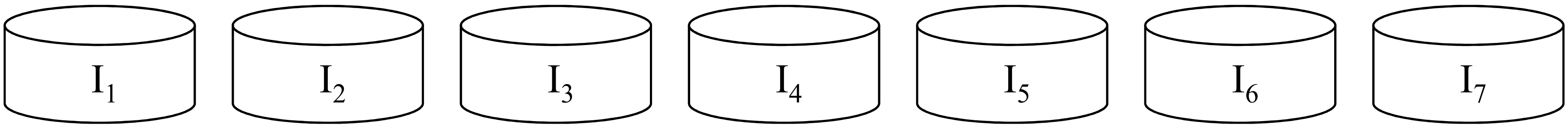
IBPS: Task Assignment



- First two phases assign tasks to k processors such that
  - each of the k processors has load greater than 55.2%

IBPS: Task Assignment

After ***Phase-1*** and ***Phase-2***, the unassigned tasks have special properties.



Empty | Number of tasks unassigned in I_2-I_6 are known | $U_7 \leq 69\%$

- These unassigned tasks are called ***residue tasks***
  - Total (unassigned) utilization is $\mathbf{U_{res}}$
- For residue tasks, the lower bound $\mathbf{U_{reslow}}$ on $\mathbf{U_{res}}$ is known
  - We have $\mathbf{U_{reslow} < U_{res}}$

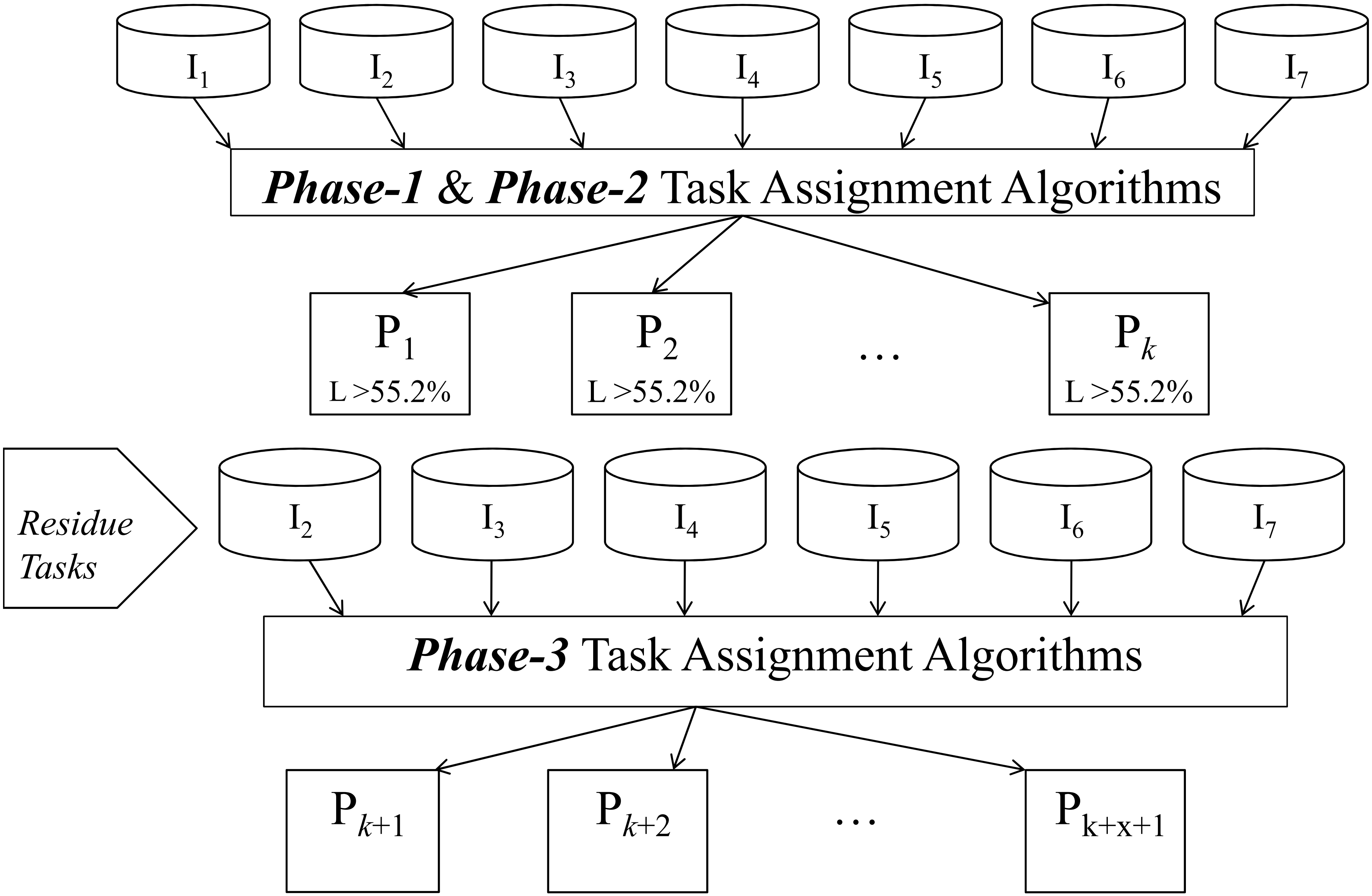
IBPS: Task Assignment-Third Phase

Given $U_{reslow} < U_{res,}$ how many processors to assign the residue tasks?

$$\mathbf{0.552\, x < U_{reslow} \leq 0.552\, (x+1) \text{ for some } x = 0, 1, 2 \ldots}$$

For example, if $U_{reslow} = 0.01$, then $x = 0$
or, if $U_{reslow} = 0.65$, then $x = 1$
or, if $U_{reslow} = 1.85$, then $x = 3$

$(x+1)$ processors are used in ***third pahse*** of task assignment to assign the residue tasks.


I1 I2 I3 I4 I5 I6 I7
Phase-1 & Phase-2 Task Assignment Algorithms
P1 L >55.2%
P2 L >55.2%
…
Pk L >55.2%
Residue Tasks
I2 I3 I4 I5 I6 I7
Phase-3 Task Assignment Algorithms
Pk+1
Pk+2
…
Pk+x+1

IBPS: Feasibility Condition

Theorem: If U(Γ) ≤ 0.552*m*, then then Γ is IBPS-schedulable on *m* processors.

Proof Sketch:

k processors are used in phase1 and phase 2
(*x*+1) processors are used in phase 3

We prove that, **If U(Γ) ≤ 0.552*m*, then (*k*+*x*+1) ≤ *m*.**

Proof Sketch (cont.):

$U_{LR} + U_{res} = U(\Gamma) \leq 0.552m$

if k processors are used in first two phases, then

$0.552\,k < U_{LR}$ *Because of Load Regulation*

if at most (x+1) processors are used in third phase, then

$0.552\,x < U_{reslow} < U_{res}$ *Because* $x0.552 < U_{reslow} \leq (x+1)0.552$

Therefore, $0.552\,k + 0.552\,x < U_{LR} + U_{res} \leq 0.552m$

Or, $k + x < m$

Or, $k + (x+1) \leq m$ *(Proved)*

IBPS and Online Scheduling

*If **U(Γ) ≤ 55.2m**, then all tasks meet deadlines on **m** processors.*

IBPS is applicable for ***online scheduling***

– If $U(\Gamma_{\mathbf{existing}}) + u_{\mathbf{new}}$ ***≤ 55.2m, then task*** τ_{new} ***is accepted.***

– *Where to assign the task?*

Online Scheduling : O-IBPS

- Load regulation $\Rightarrow$ third phase requires at most 4 processors (i.e. $x+1 \leq 4$)

- Load regulation enables efficient online scheduling
  - When a task arrives, tasks are reassigned to at most ***min(m, 4)*** processors
  - When a task leaves, tasks are reassigned to at most ***min(m, 5)*** processors

- Therefore, *O-IBPS* ***scales very well*** for large systems.

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Conclusion

- *IBPS has many advantages in comparison to other task-splitting algorithms*
  - —utilization bound of 55.2%
  - —*load-regulation*
    - — *online scheduling*
    - —*scalable*
  - — *low overhead of task splitting*
    - — Only *m*/2 split tasks.

Thank You