

# Efficient Floating-Point Logarithm Unit for FPGAs

**Nikolaos Alachiotis, Alexandros Stamatakis**

The Exelixis Lab,  
Dept. of Computer Science,  
TUM, Munich, Germany

# PRESENTATION OVERVIEW

- Introduction
- Approximation Strategy
- Reconfigurable Architecture
- Performance Evaluation
- Conclusion and Future Work

- The Project:

Design of HW accelerators for Phylogenetic Inference Programs

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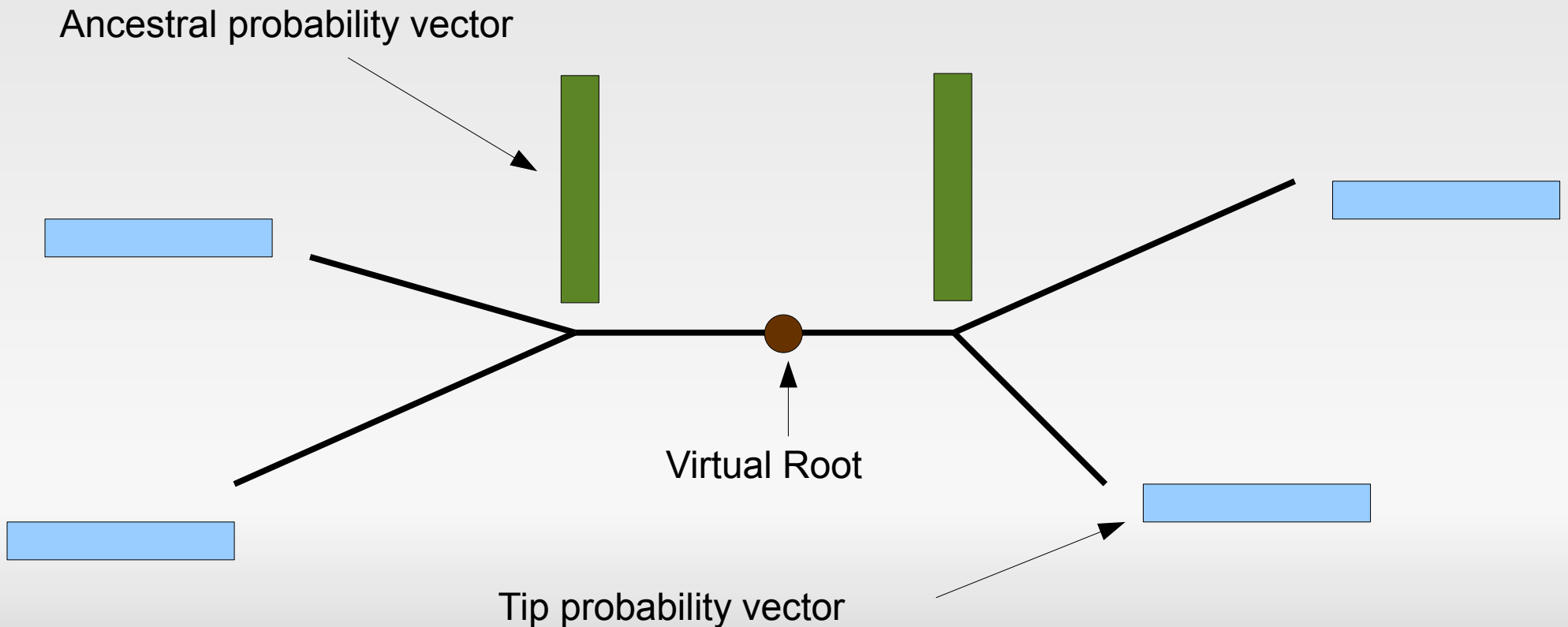


Calculation of evolutionary relationships between organisms

core function: the Phylogenetic Likelihood Function

- The Project:

## Design of HW accelerators for Phylogenetic Inference Programs



- The Project:

Design of HW accelerators for Phylogenetic Inference Programs

- The Phylogenetic Likelihood Function:

85% of total execution time

- Log-Likelihood Scores:

2% of total execution time

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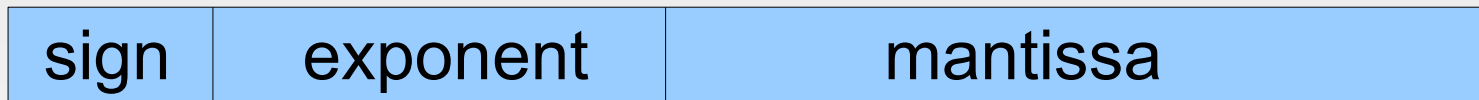
**Need for a  
resource-efficient  
logarithm function**

# APPROXIMATION STRATEGY

“A Hardware-Independent Fast Logarithm Approximation with Adjustable Accuracy,” by O. Vinyals, G. Friedland.  
*Tenth IEEE Inter. Symposium on Multimedia*, pp. 61–65, 2008.

Open source C implementation: ICSILog 0.6 BETA

Floating-Point number in IEEE-754 standard



$$\text{Number} = \text{sign} * 2^{\text{exponent}} * \text{mantissa}$$



# APPROXIMATION STRATEGY

$$\text{Number} = \text{sign} * 2^{\text{exponent}} * \text{mantissa}$$

Logarithm defined only for positive values

$$\text{LOG}(\text{Number}) = \text{LOG} ( 2^{\text{exponent}} * \text{mantissa} )$$

Multiplicative property of logarithm

$$= \text{LOG} ( 2^{\text{exponent}} ) + \text{LOG}(\text{mantissa})$$

$$= \text{exponent} * \text{LOG} (2) + \text{LOG}(\text{mantissa})$$

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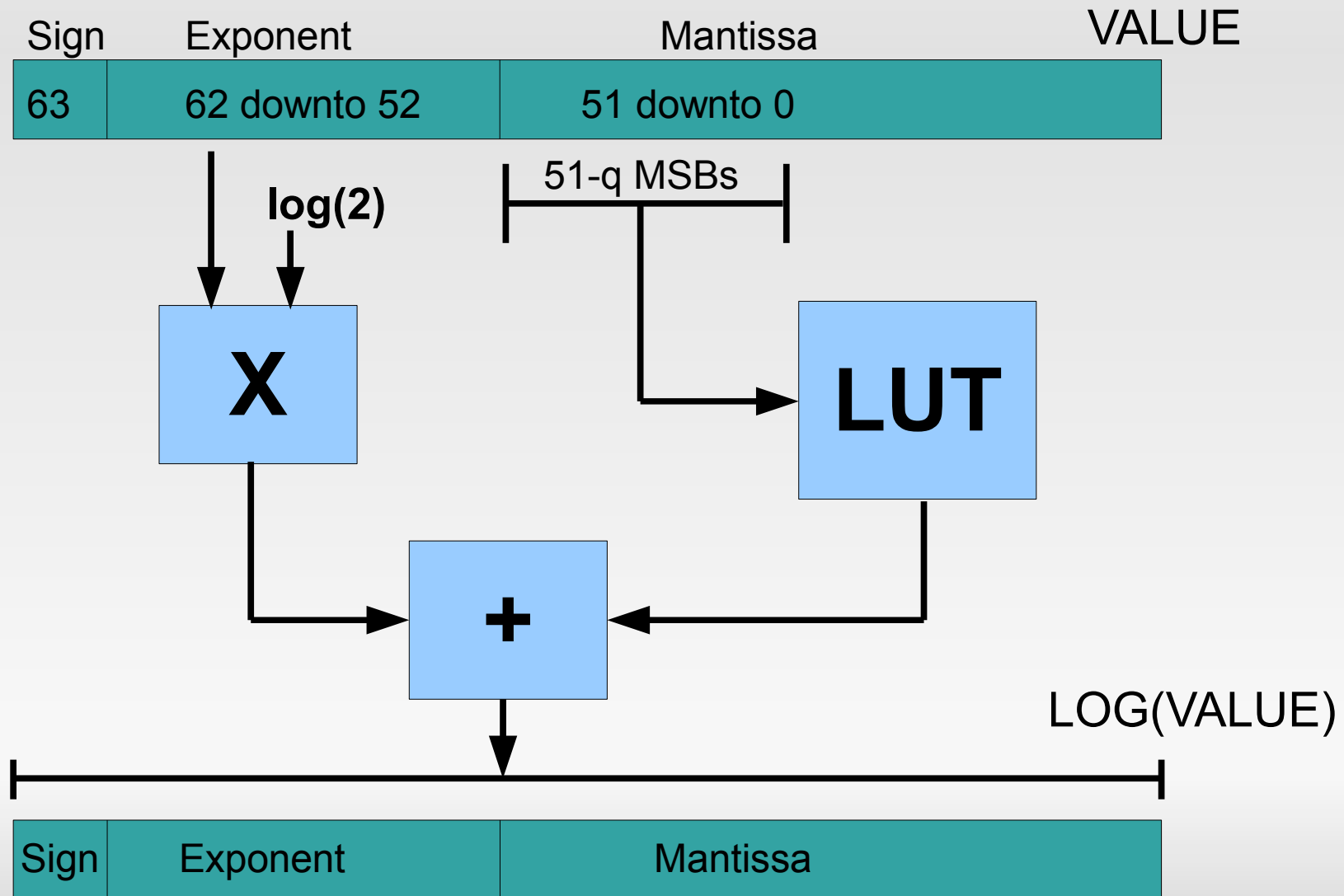
$$= \text{LOG} ( 2^{\text{exponent}} ) + \text{LOG}(\text{mantissa})$$

$$= \text{exponent} * \text{LOG} (2) + \text{LOG}(\text{mantissa})$$

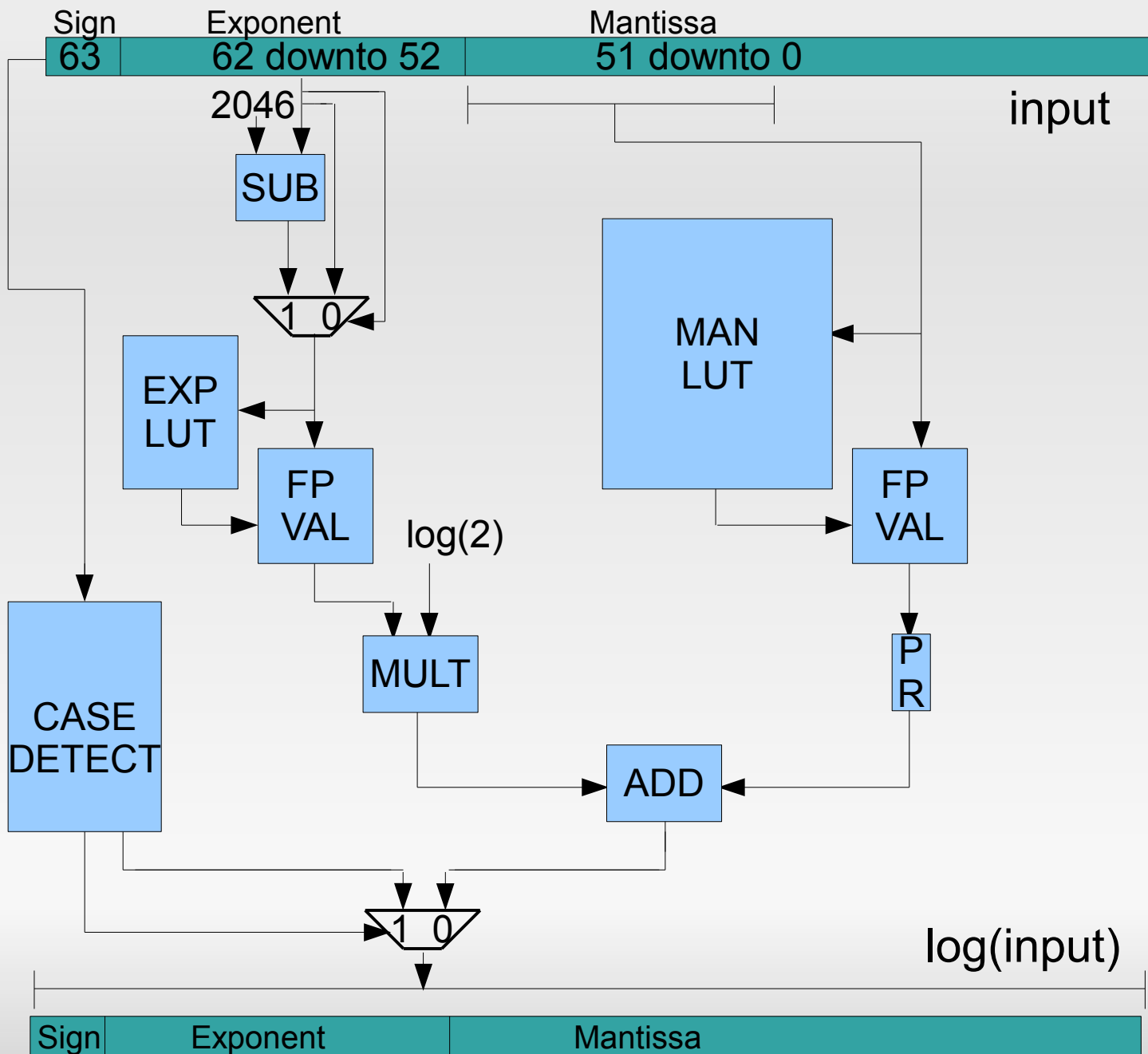
Lookup Table

# APPROXIMATION STRATEGY

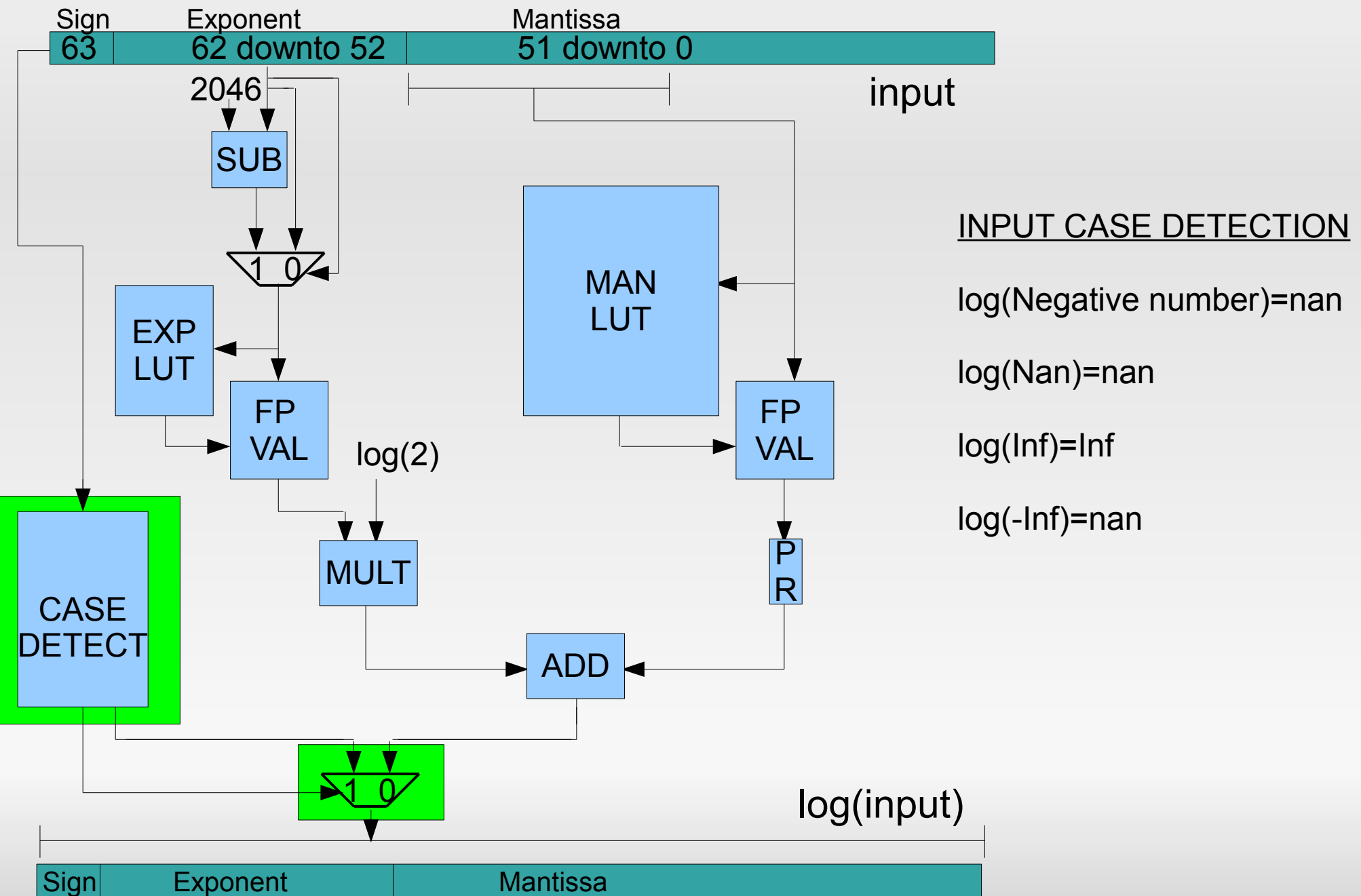
$$\text{LOG}(\text{Value}) = \text{exponent} * \text{LOG}(2) + \text{LOG}(\text{mantissa})$$



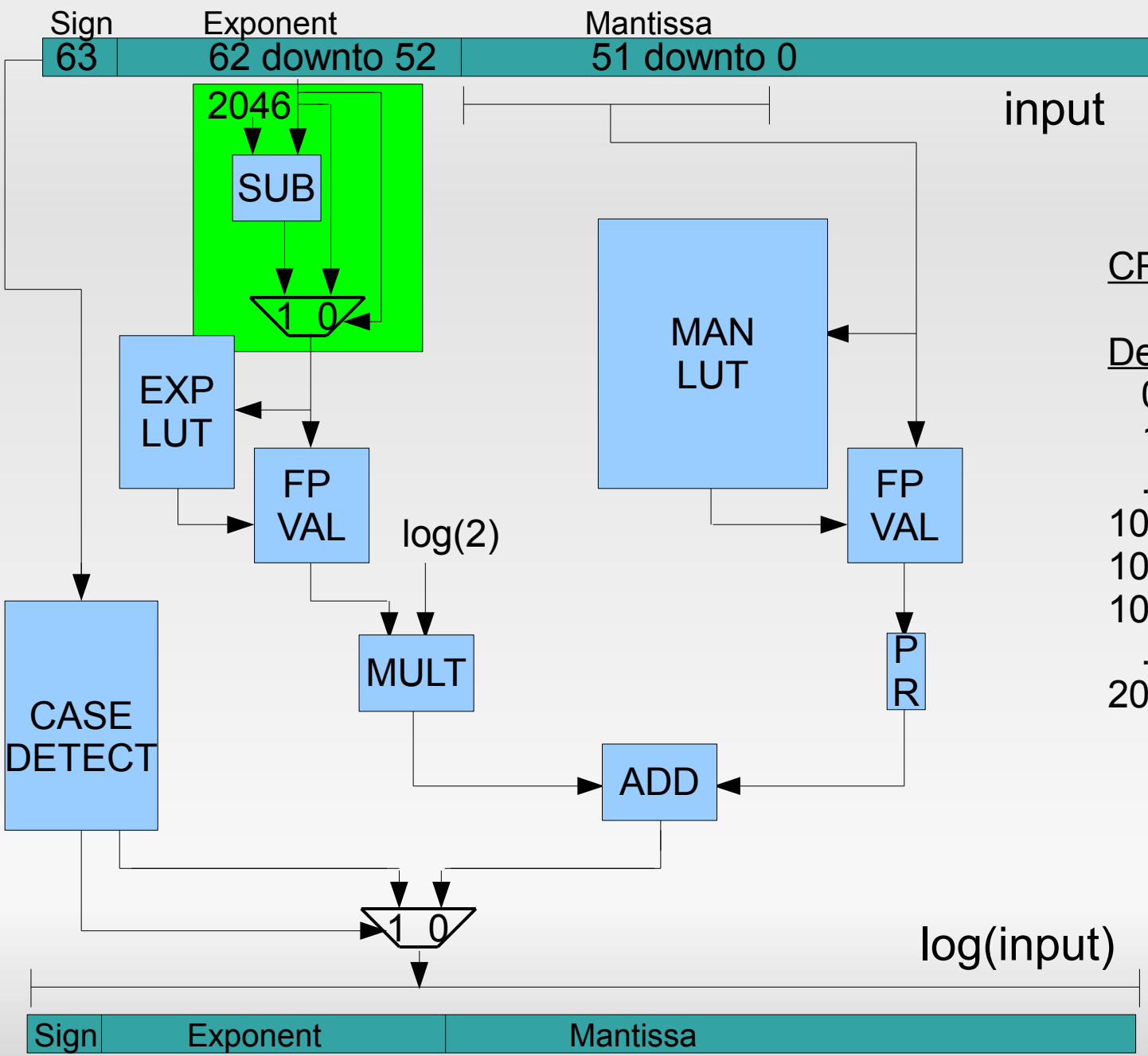
# LOGARITHM APPROXIMATION UNIT (LAU) ARCHITECTURE



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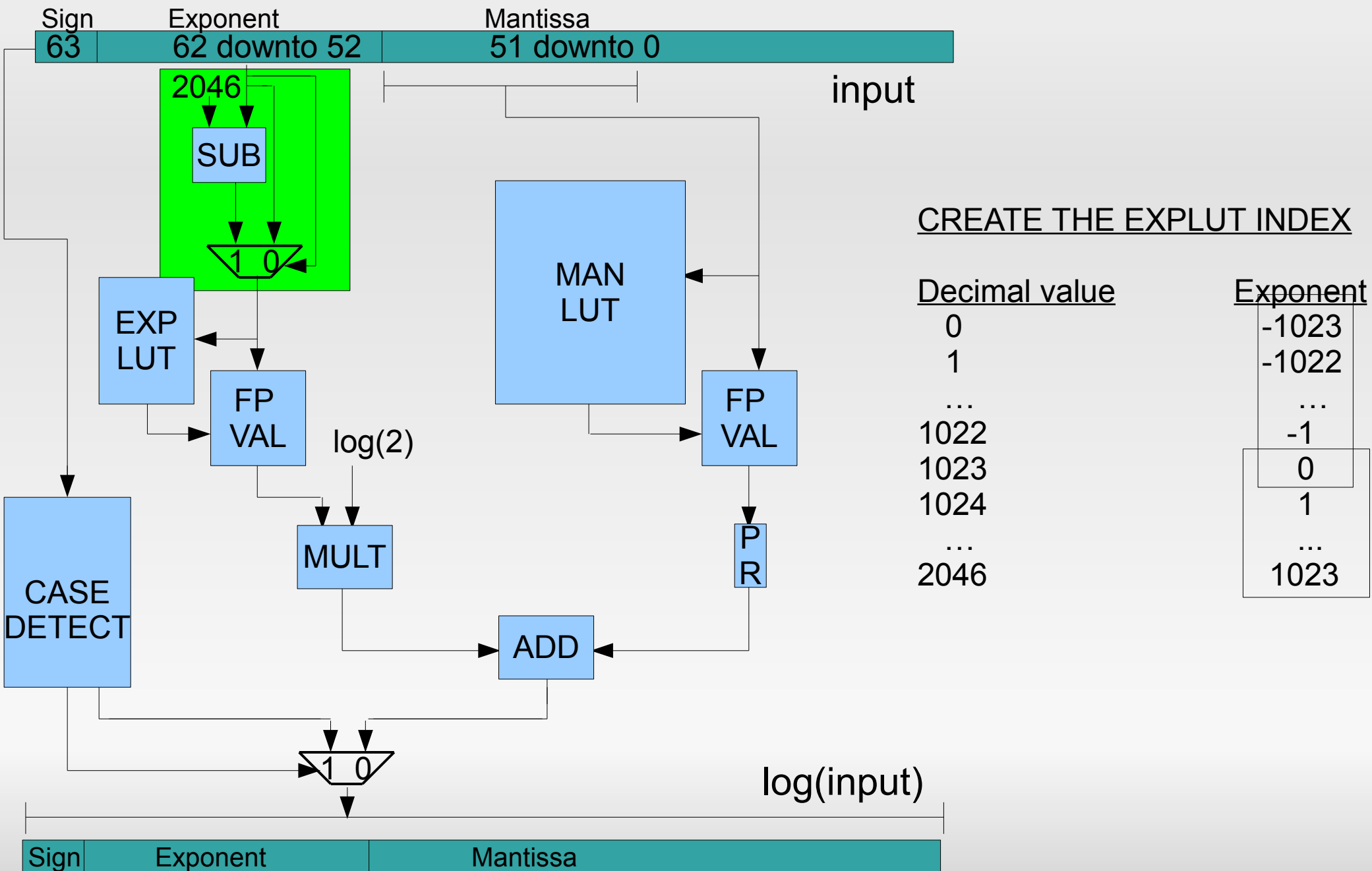
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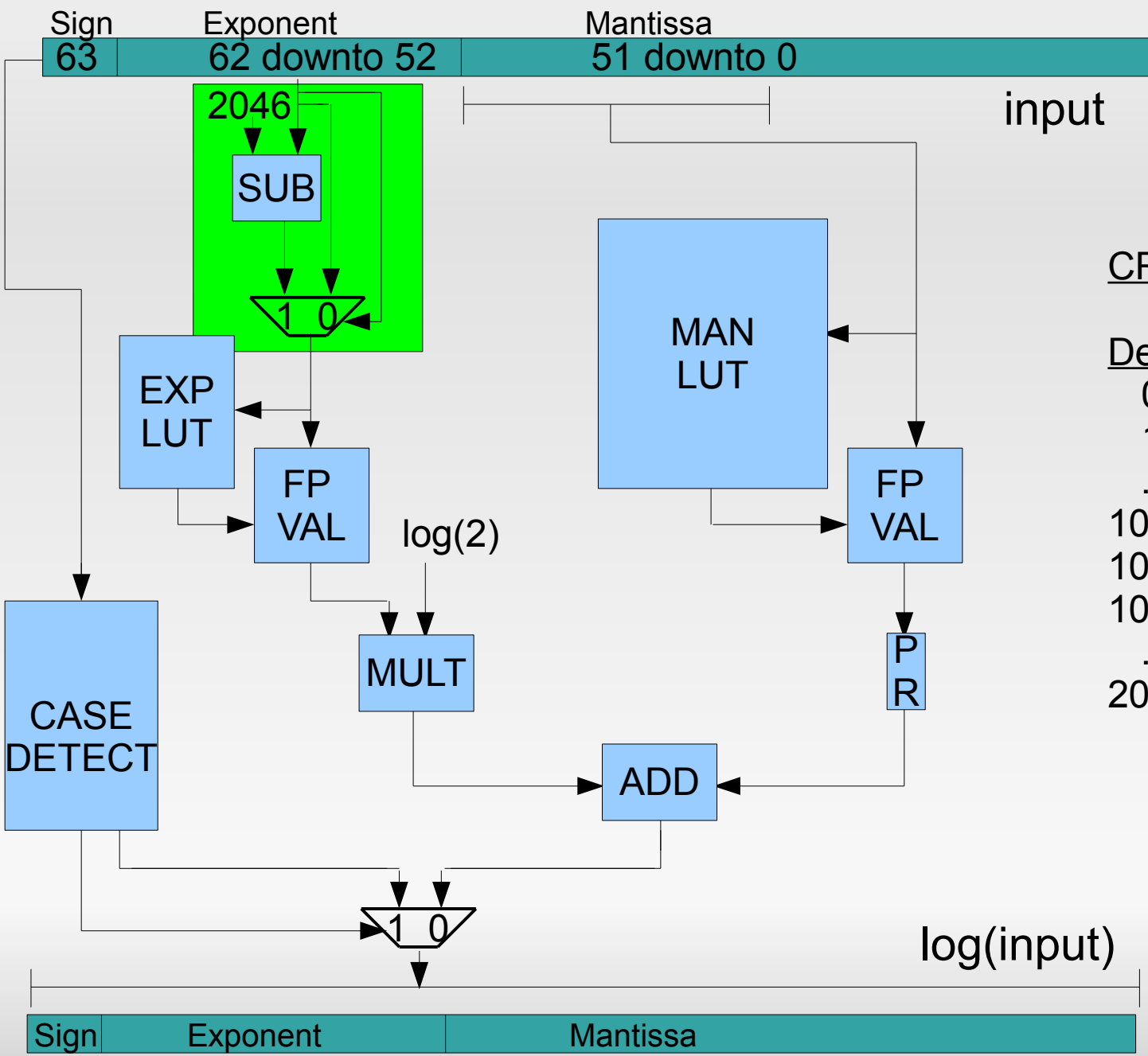
CREATE THE EXPLUT INDEX

<u>Decimal value</u>	<u>Exponent</u>
0	-1023
1	-1022
...	...
1022	-1
1023	0
1024	1
...	...
2046	1023

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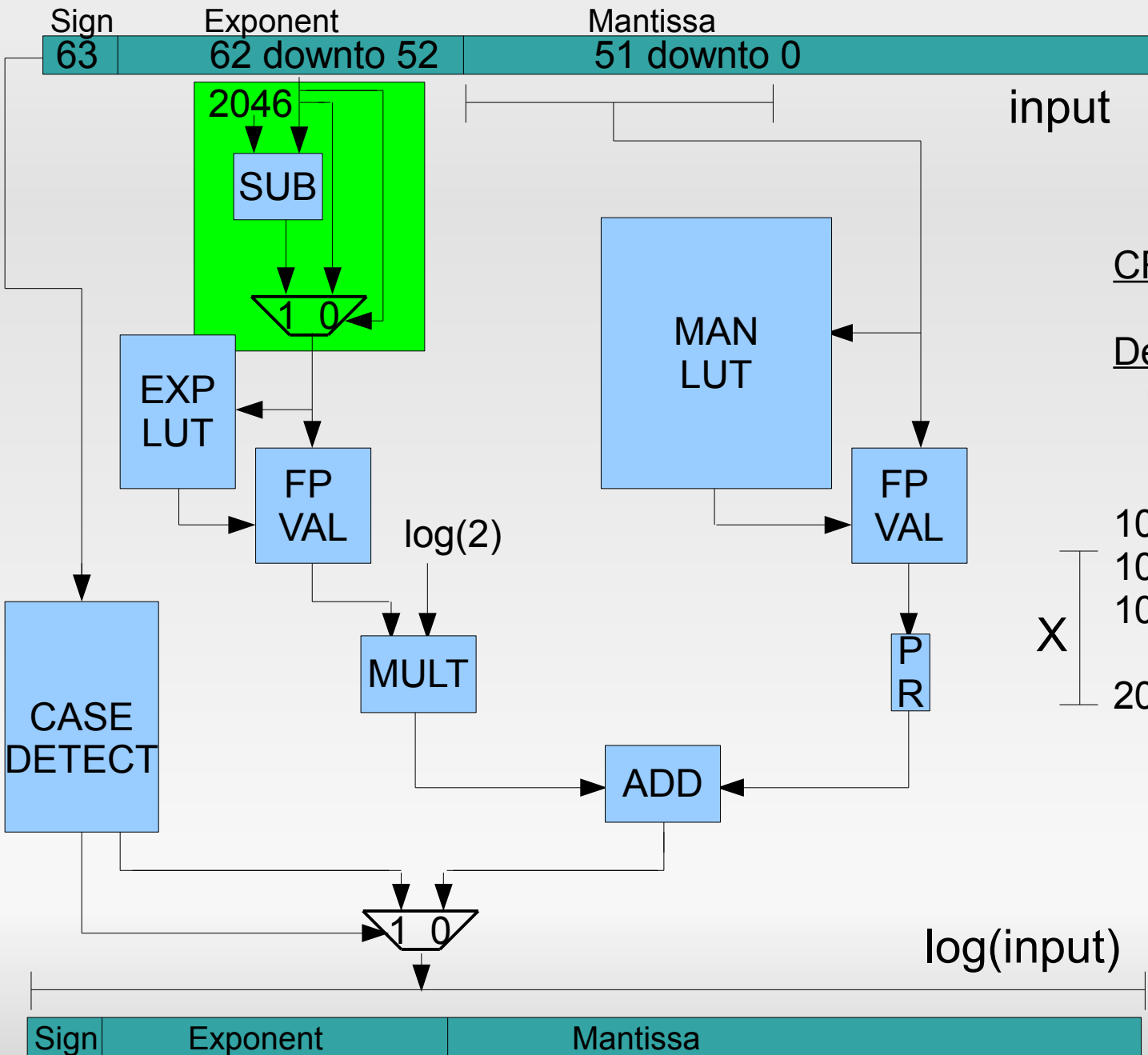


CREATE THE EXPLUT INDEX

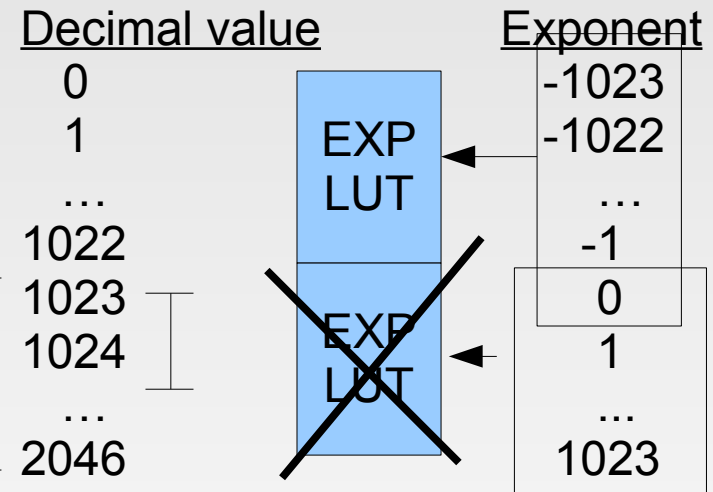
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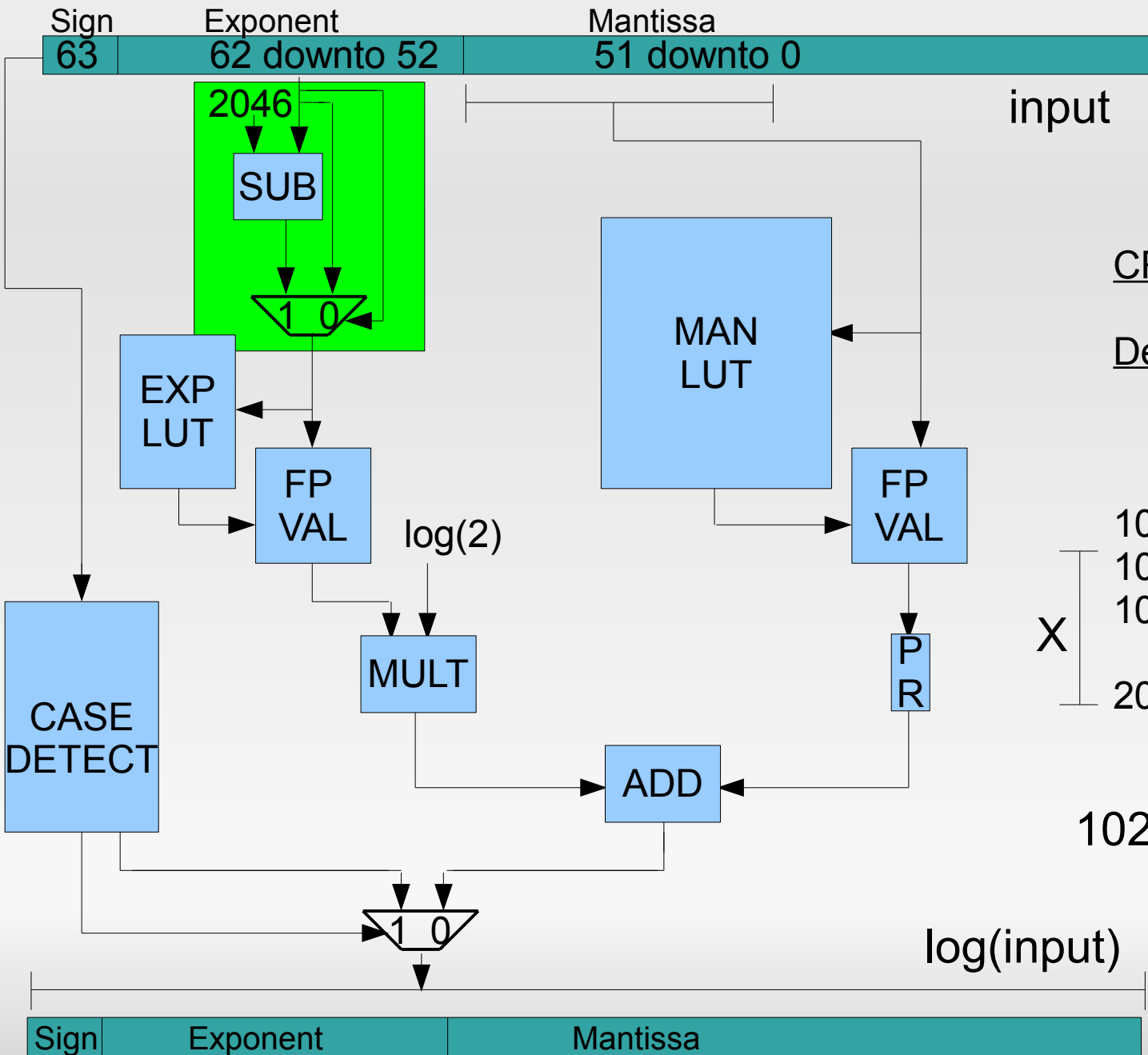


CREATE THE EXPLUT INDEX

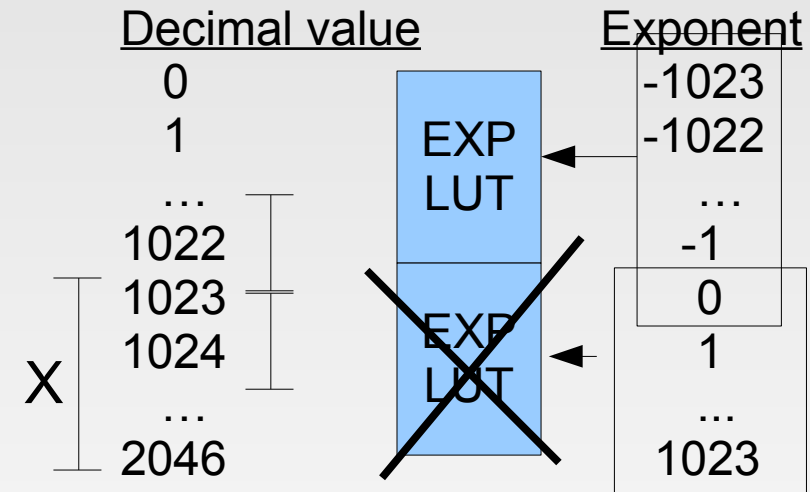


$X - 1023$

# LOGARITHM APPROXIMATION UNIT (LAU) ARCHITECTURE

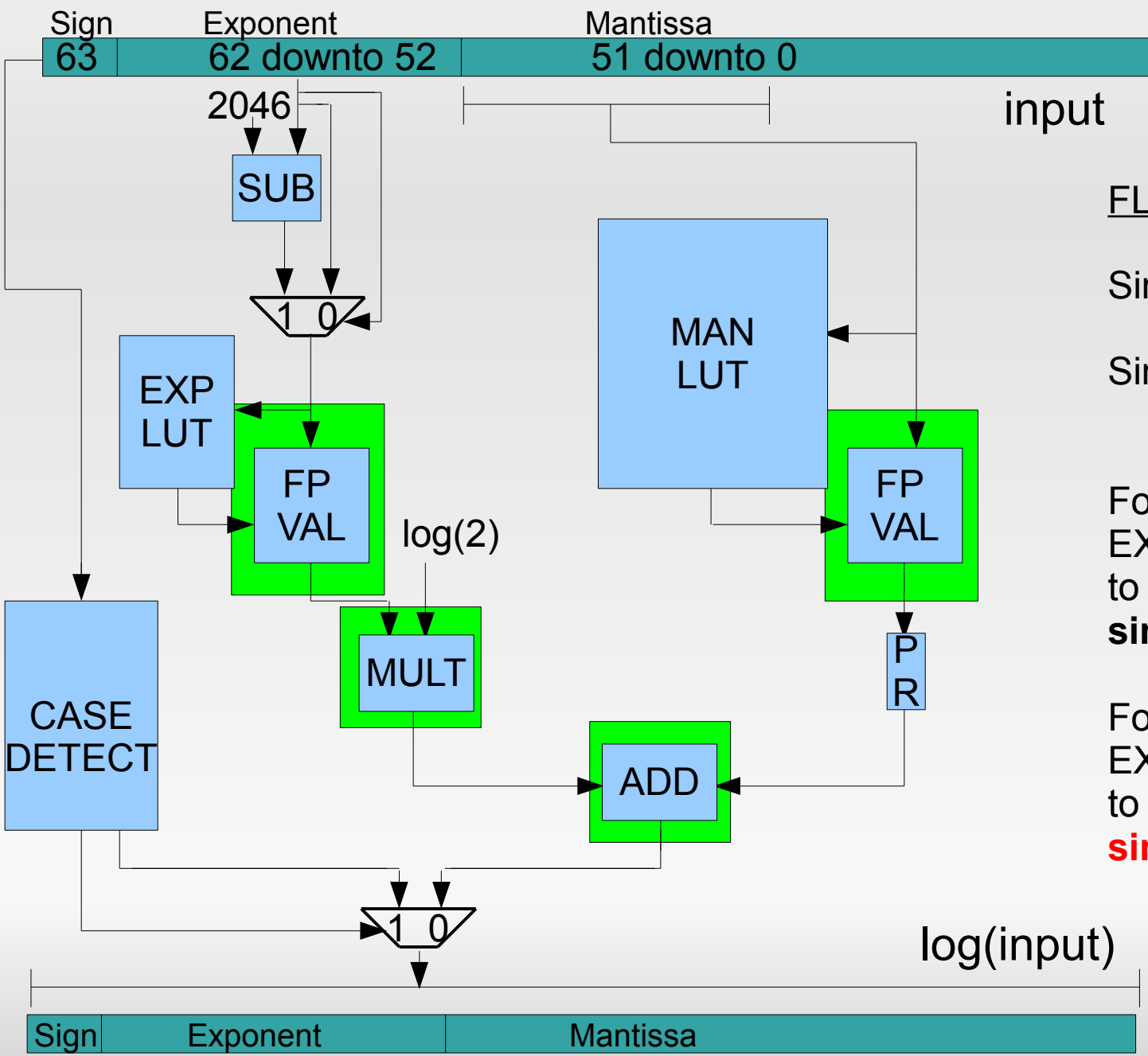


CREATE THE EXPLUT INDEX



$$1023 - (X - 1023) = 2046 - X$$

# LOGARITHM APPROXIMATION UNIT (LAU) ARCHITECTURE



FLOATING-POINT VALUE

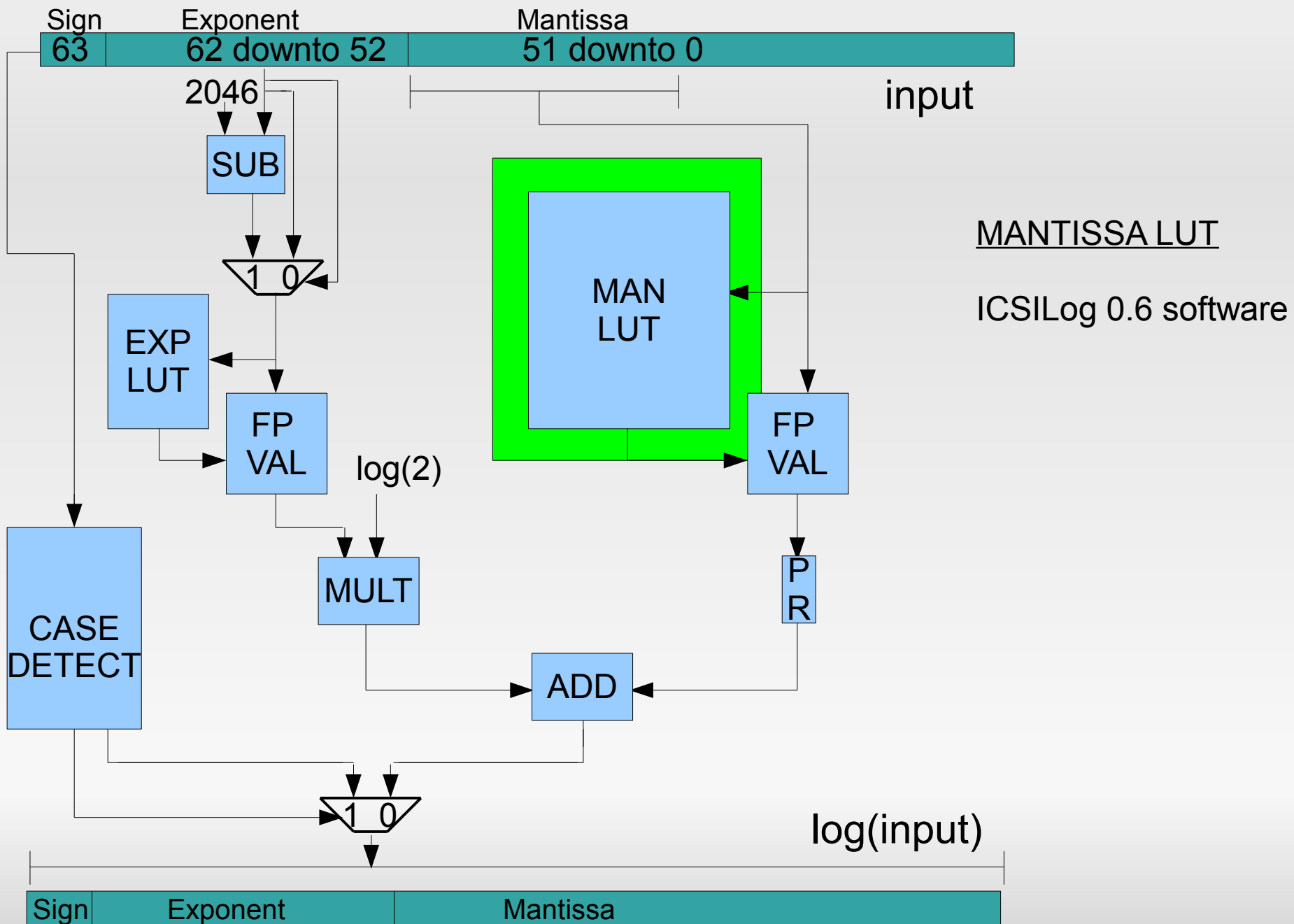
Single-precision values

Single-precision MULT and ADD

For **single-precision** inputs  
EXPLUT contains 128 entries  
to construct a  
**single-precision** value

For **double-precision** inputs  
EXPLUT contains 1024 entries  
to construct a  
**single-precision** value

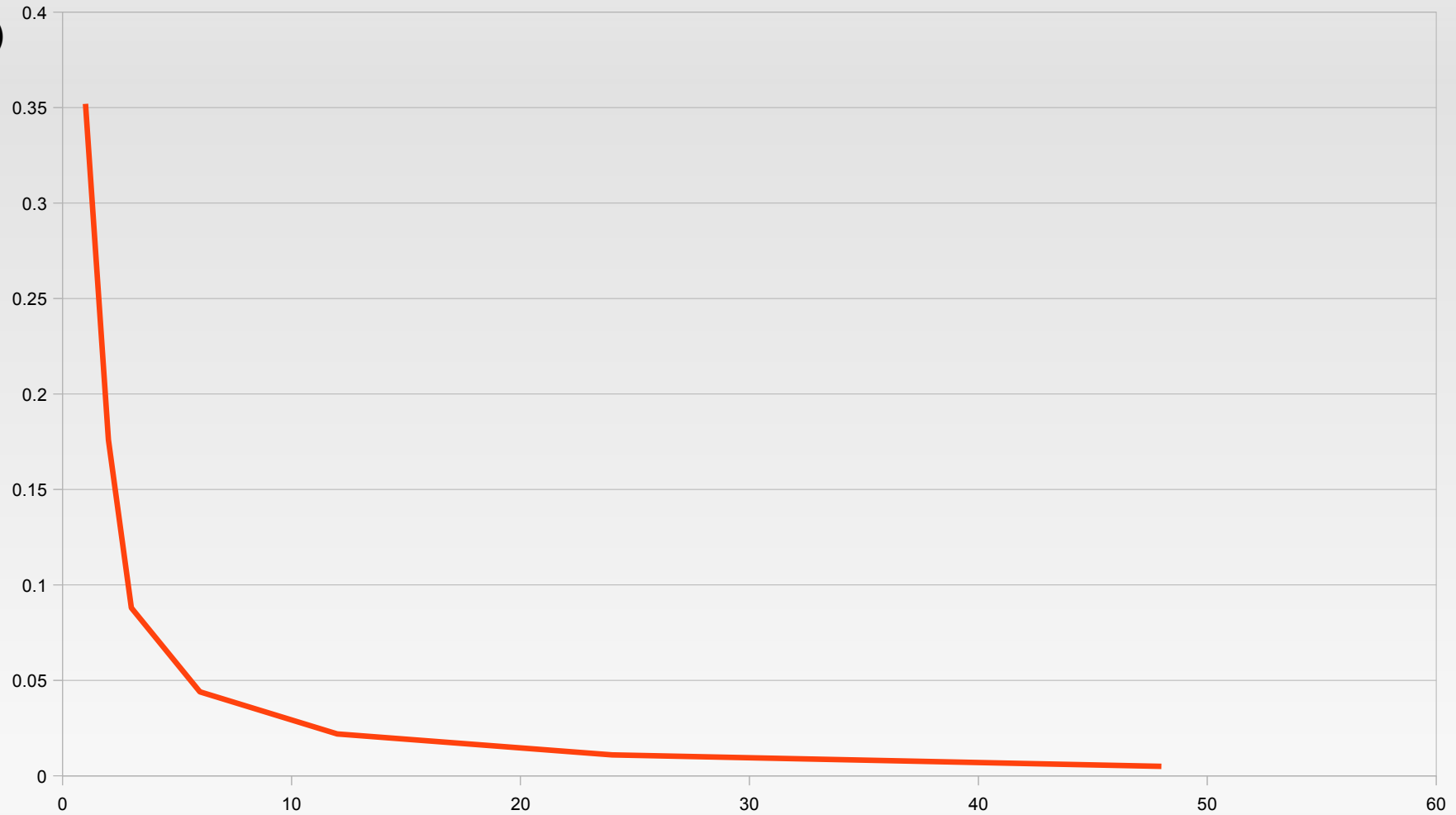
# LOGARITHM APPROXIMATION UNIT (LAU) ARCHITECTURE



# PERFORMANCE EVALUATION

## Accuracy Versus Hardware resources

Average  
Error  
( $\times 10^3$ )

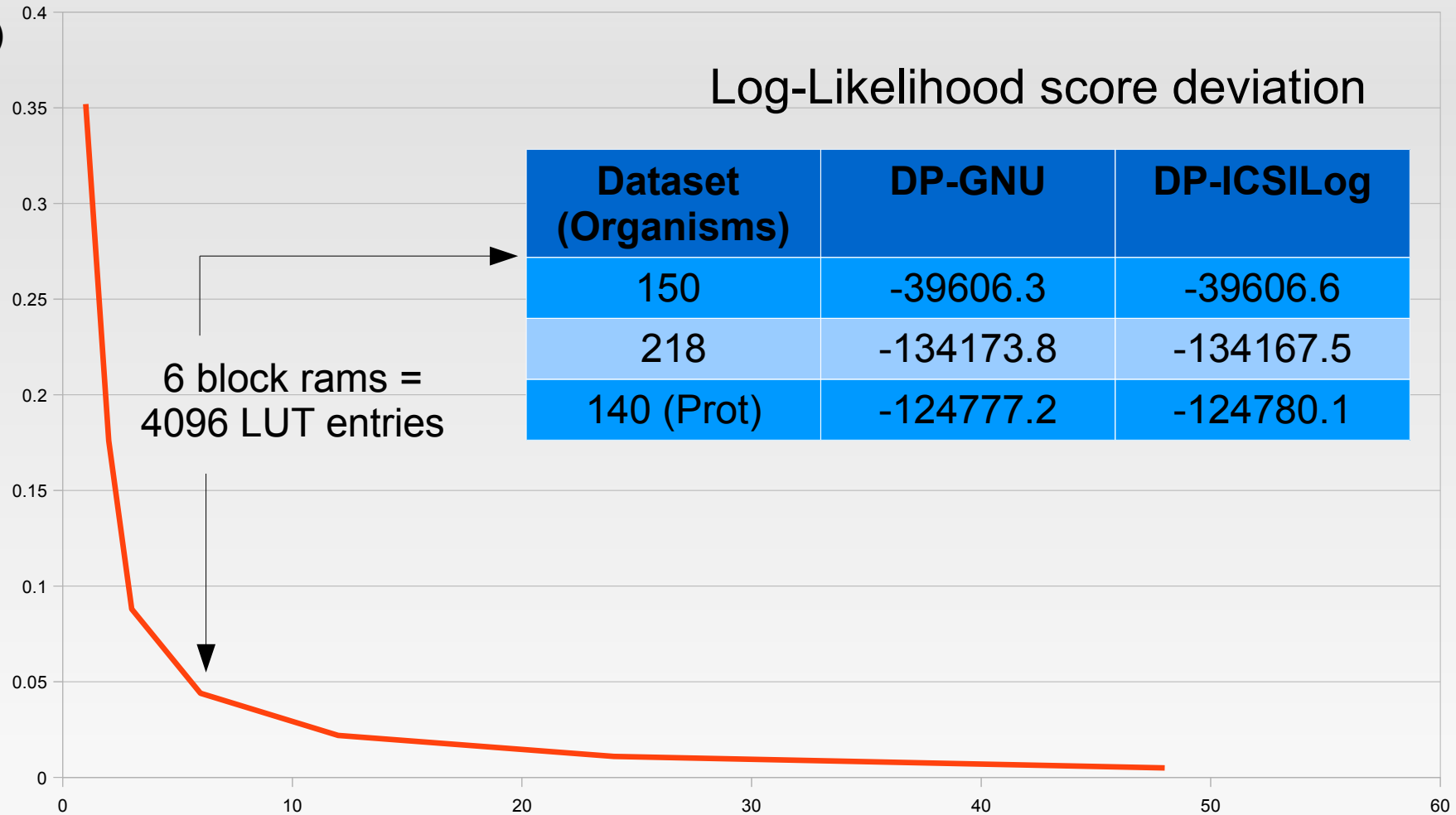


Resources (Number of 18Kb block rams)

# PERFORMANCE EVALUATION

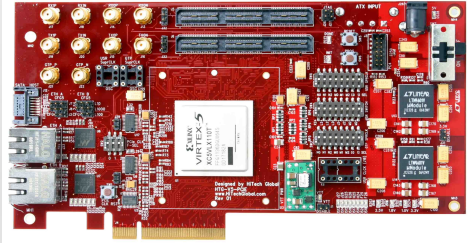
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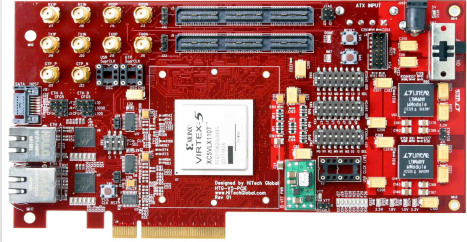
VIRTEX 5 SX95T for mapping and verification

XILINX ISE 10.1 and CHIPSCOPE Pro Analyzer

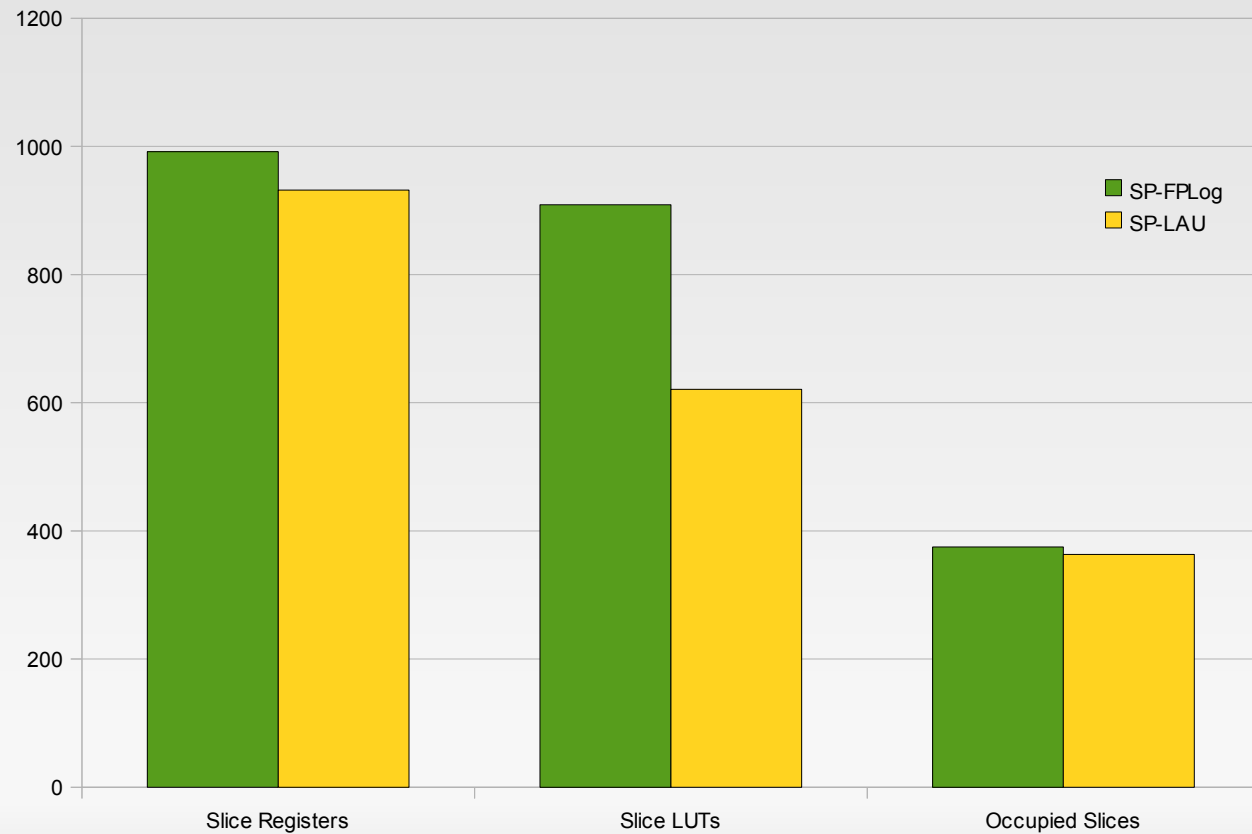
F. de Dinechin, C. Klein, B. Pasca,

“Generating high-performance custom floating-point pipelines,” *Proc. of FPL 2009*.

# PERFORMANCE EVALUATION

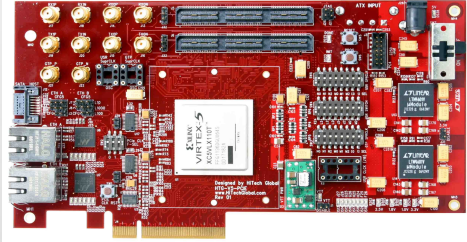


## Resource Utilization and Performance: Single Precision

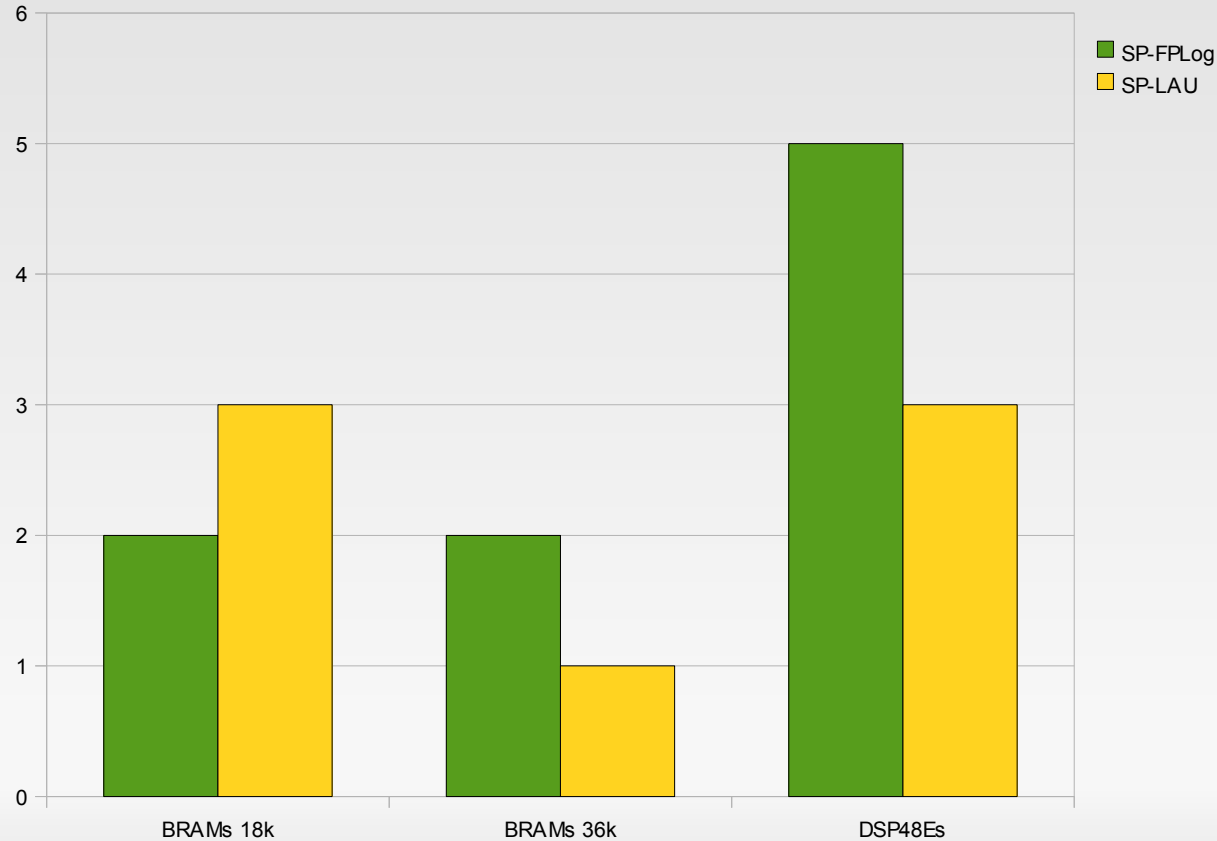




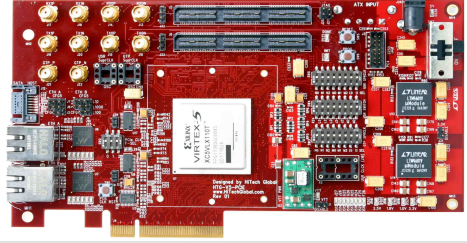
# PERFORMANCE EVALUATION



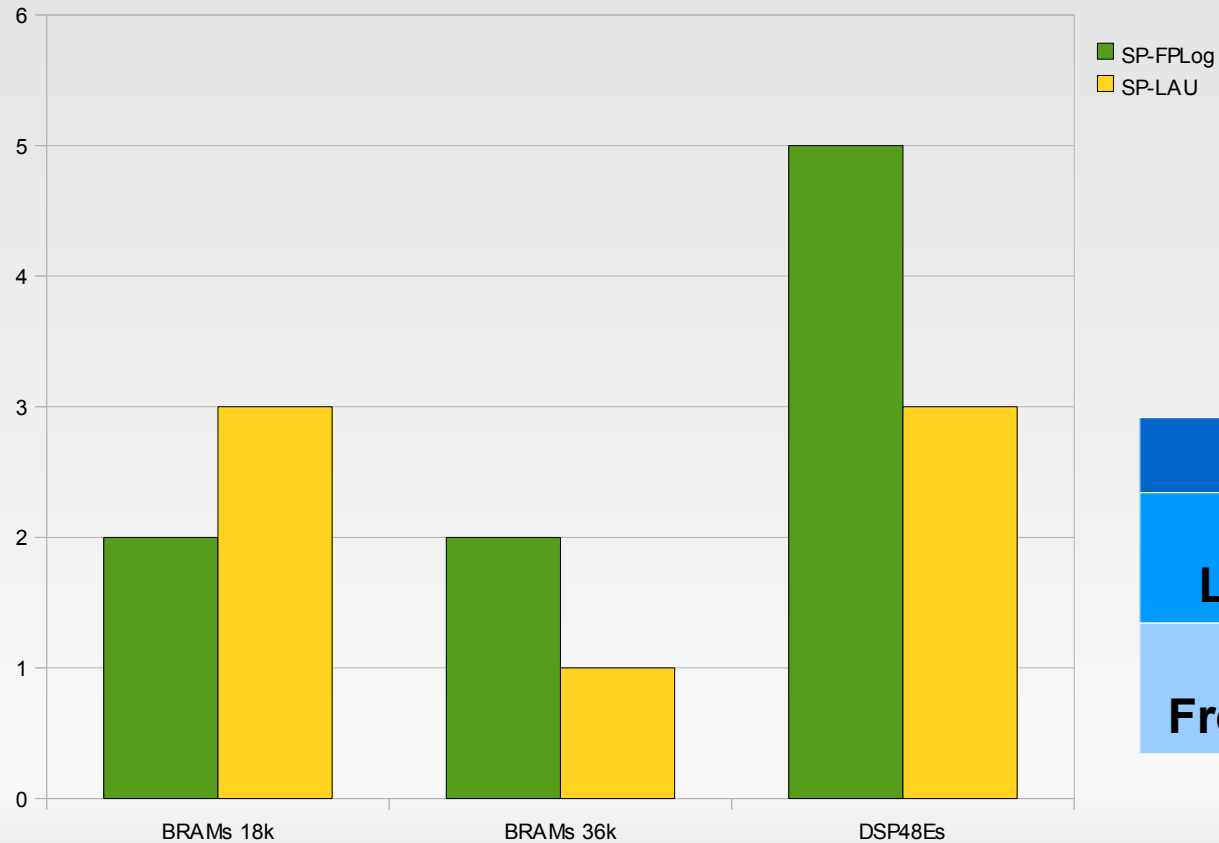
## Resource Utilization and Performance: Single Precision



# PERFORMANCE EVALUATION

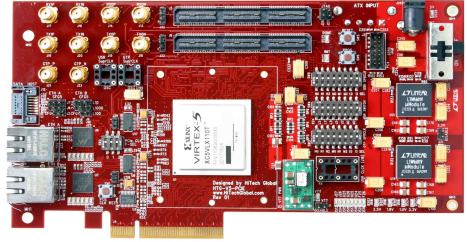


## Resource Utilization and Performance: Single Precision

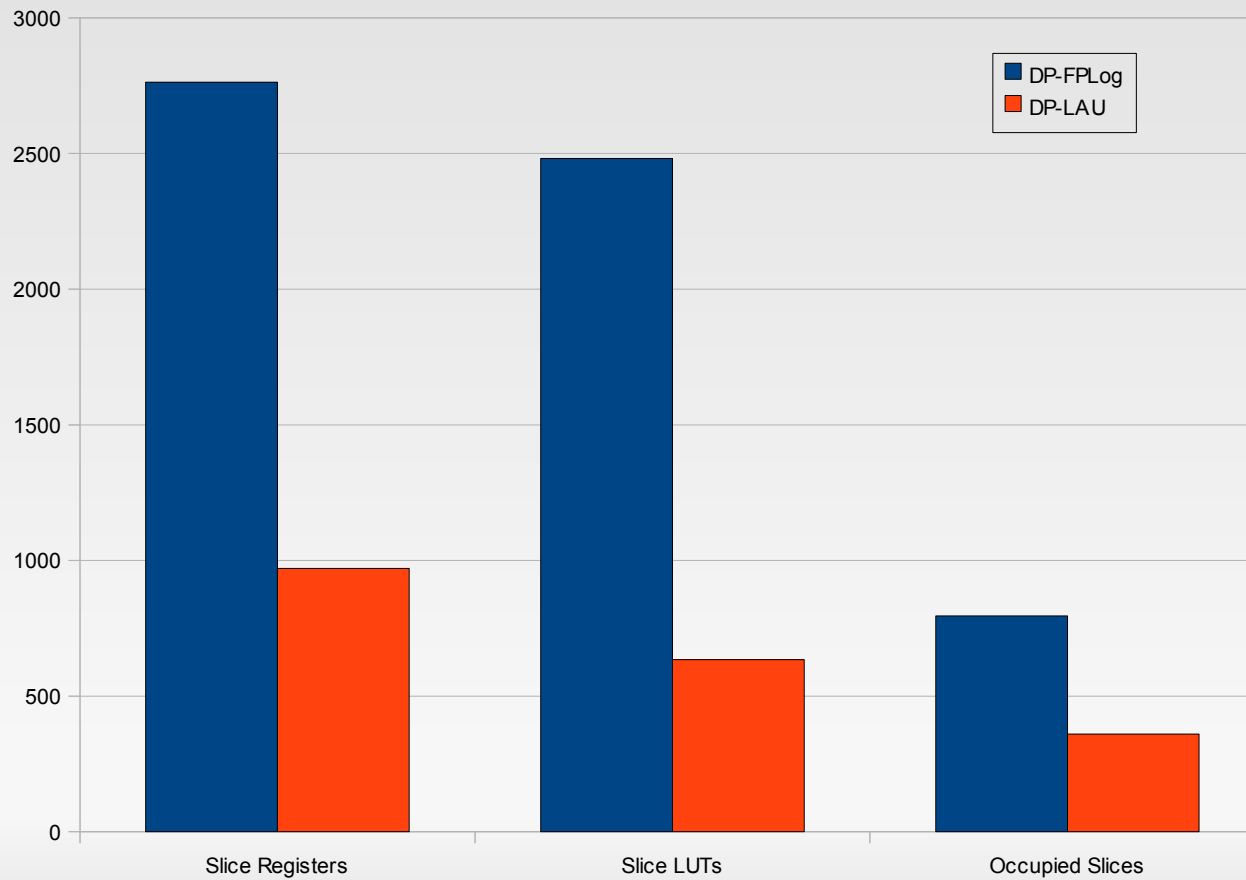


	FPLo	LAU
<b>Clock Latency</b>	20	22
<b>Max Frequency</b>	244.7	353.5

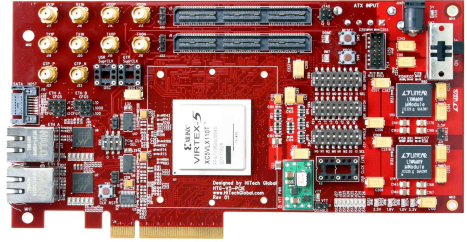
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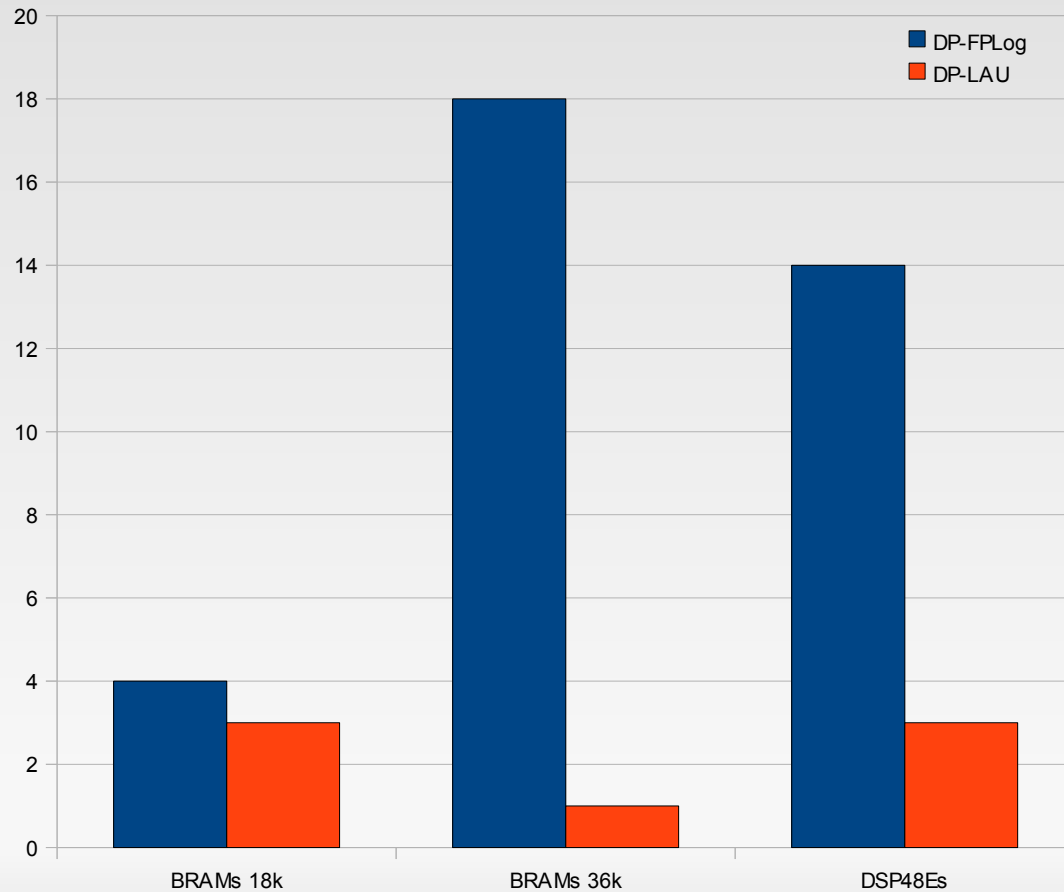
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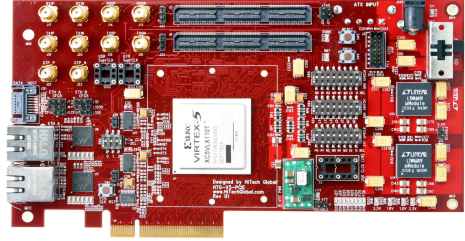
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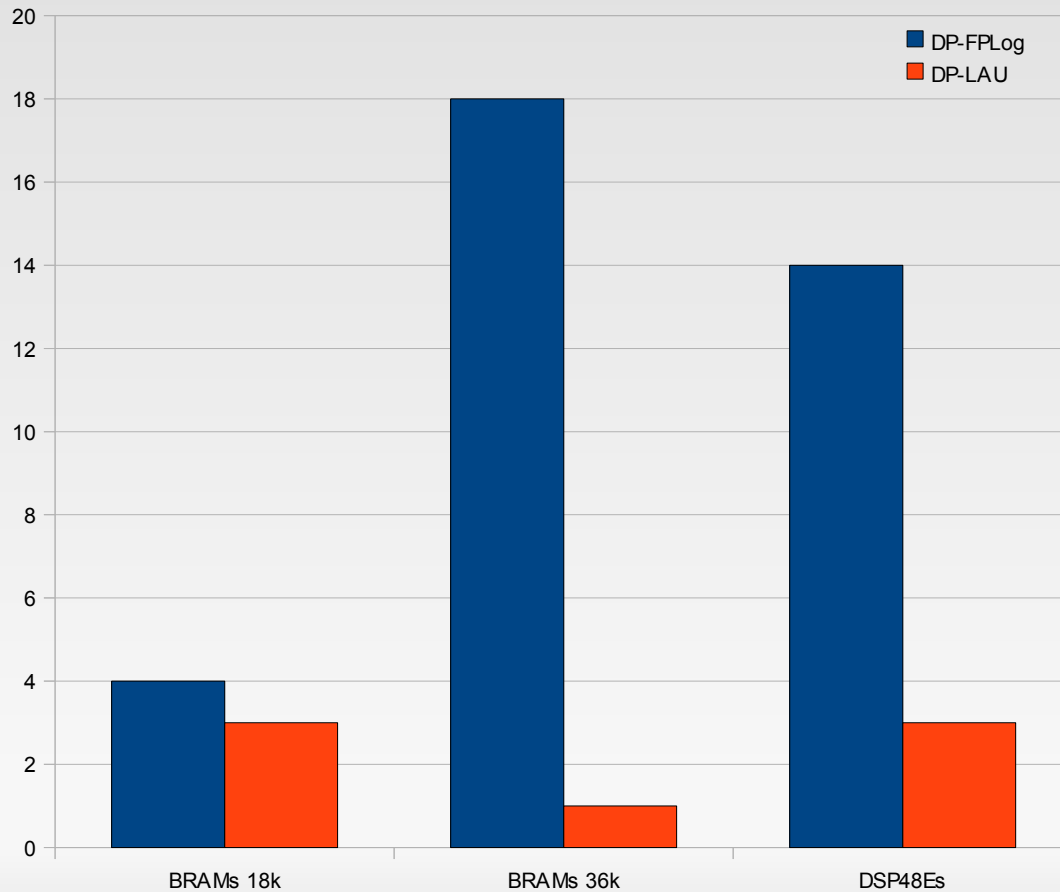
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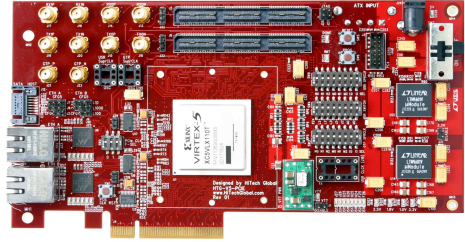


## Resource Utilization and Performance: Double Precision



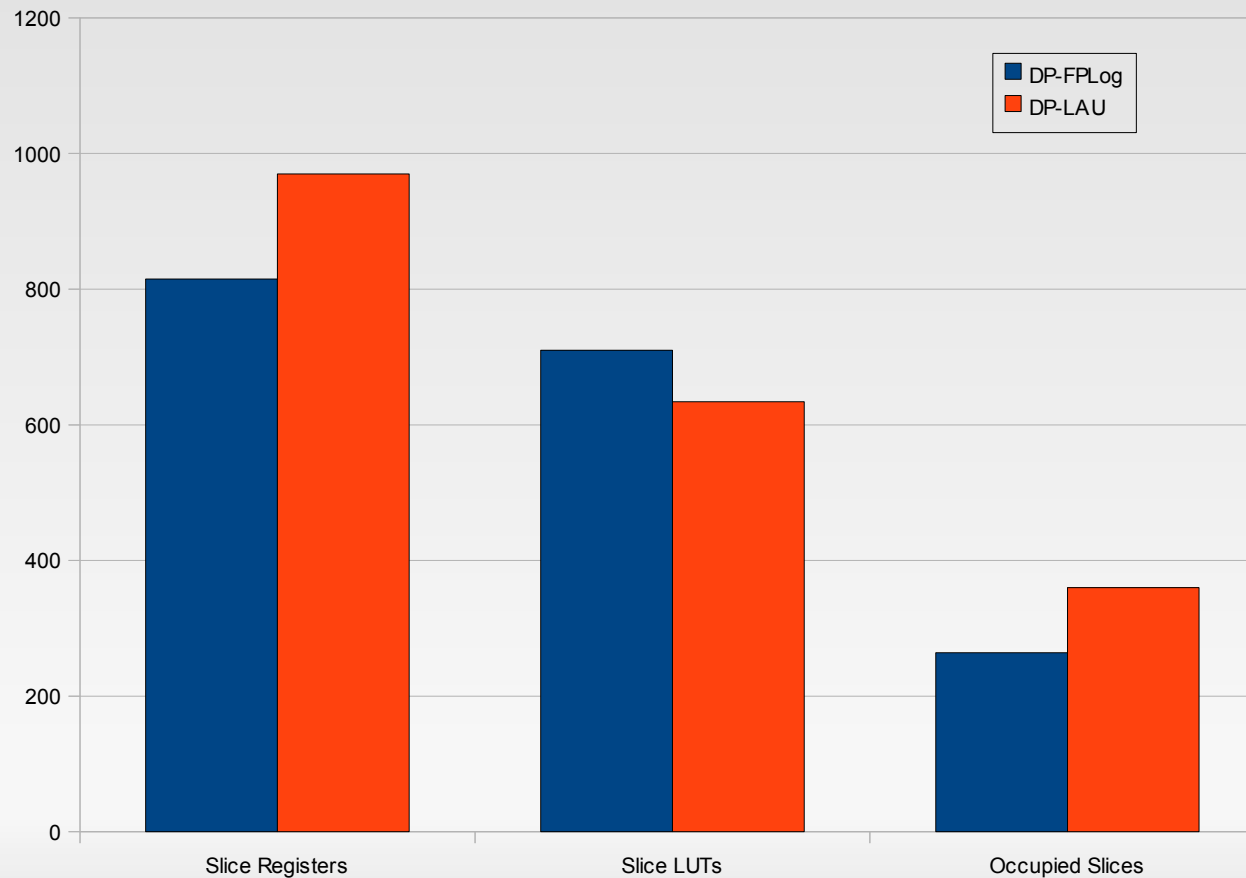
	FPLog	LAU
Clock Latency	34	22
Max Frequency	192.3	320.6

# PERFORMANCE EVALUATION

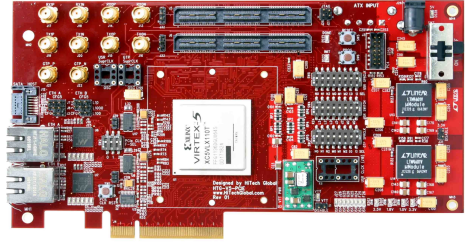


## Resource Utilization and Performance: Double Precision

DP-FPLog with same accuracy as DP-LAU

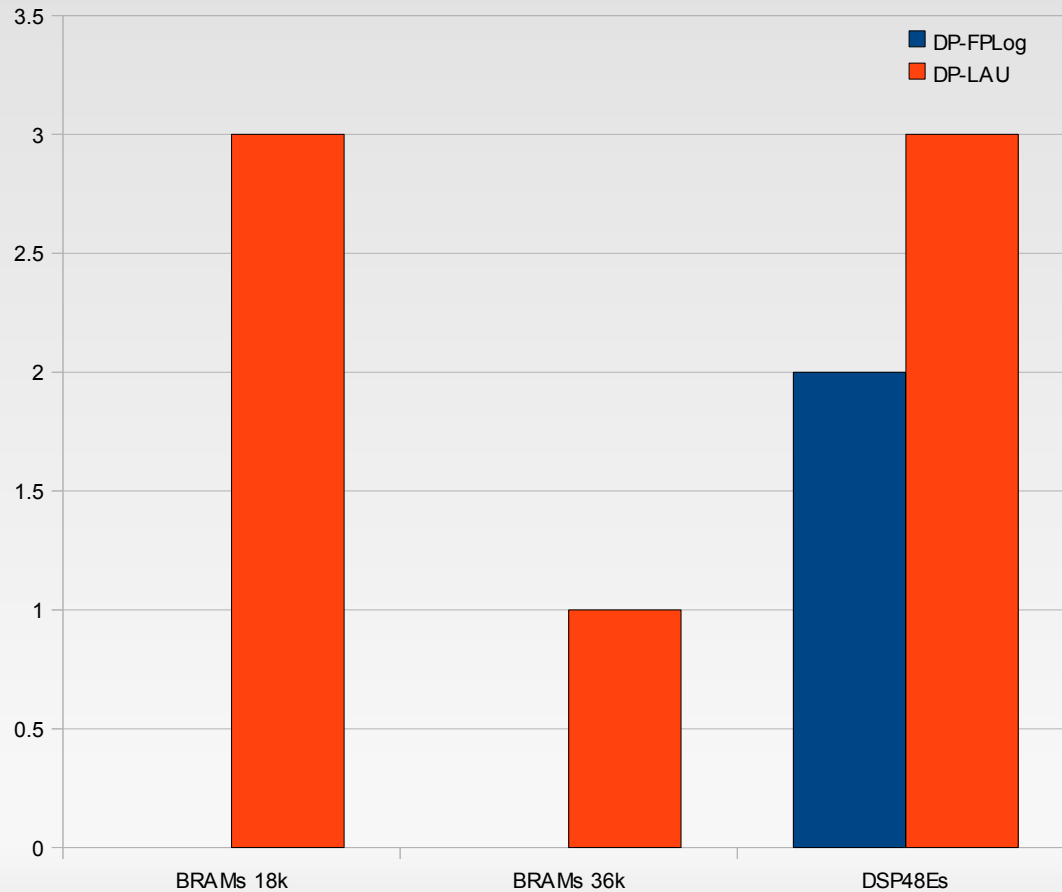


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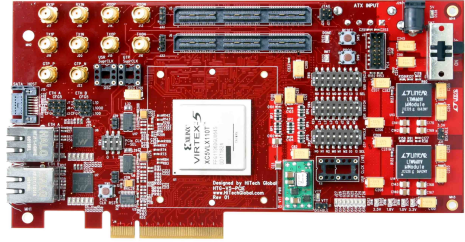


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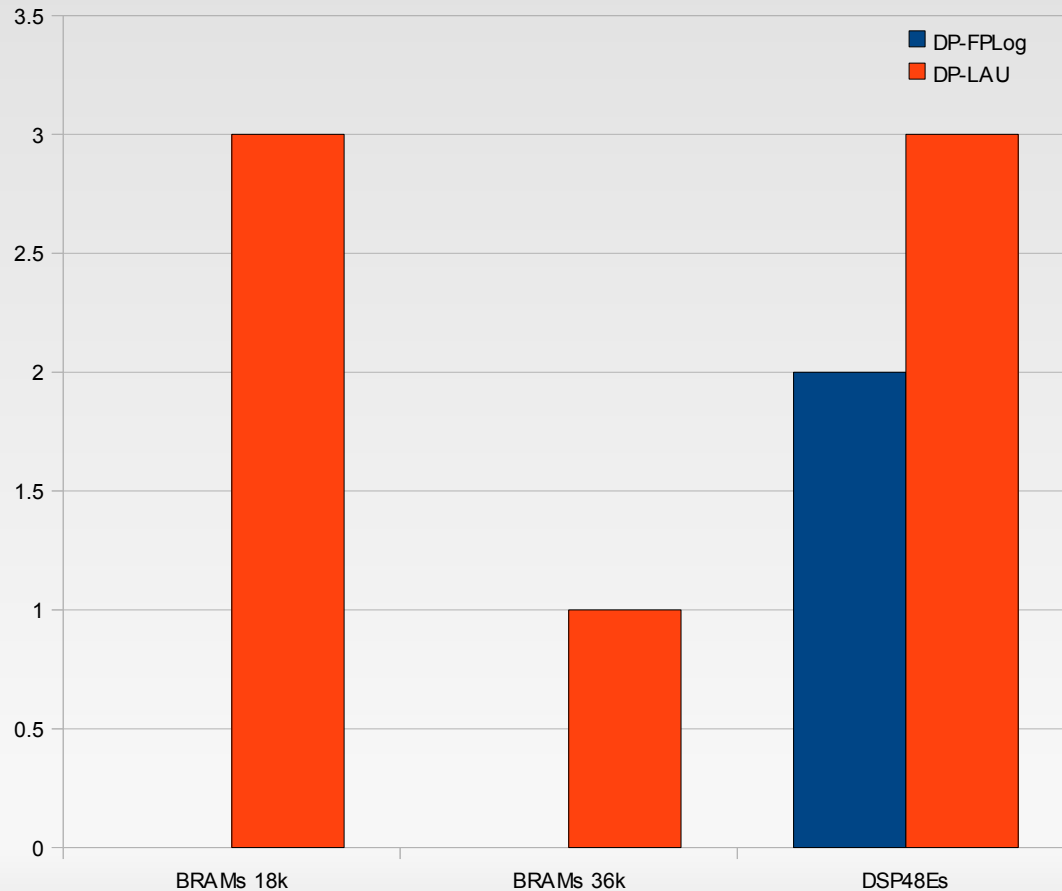


# PERFORMANCE EVALUATION



## Resource Utilization and Performance: Double Precision

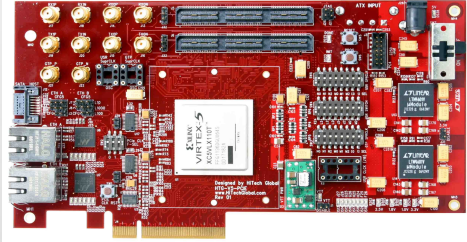
DP-FPLog with same accuracy as DP-LAU



	FPLog	LAU
Clock Latency	20	22
Max Frequency	239.6	320.6



# PERFORMANCE EVALUATION

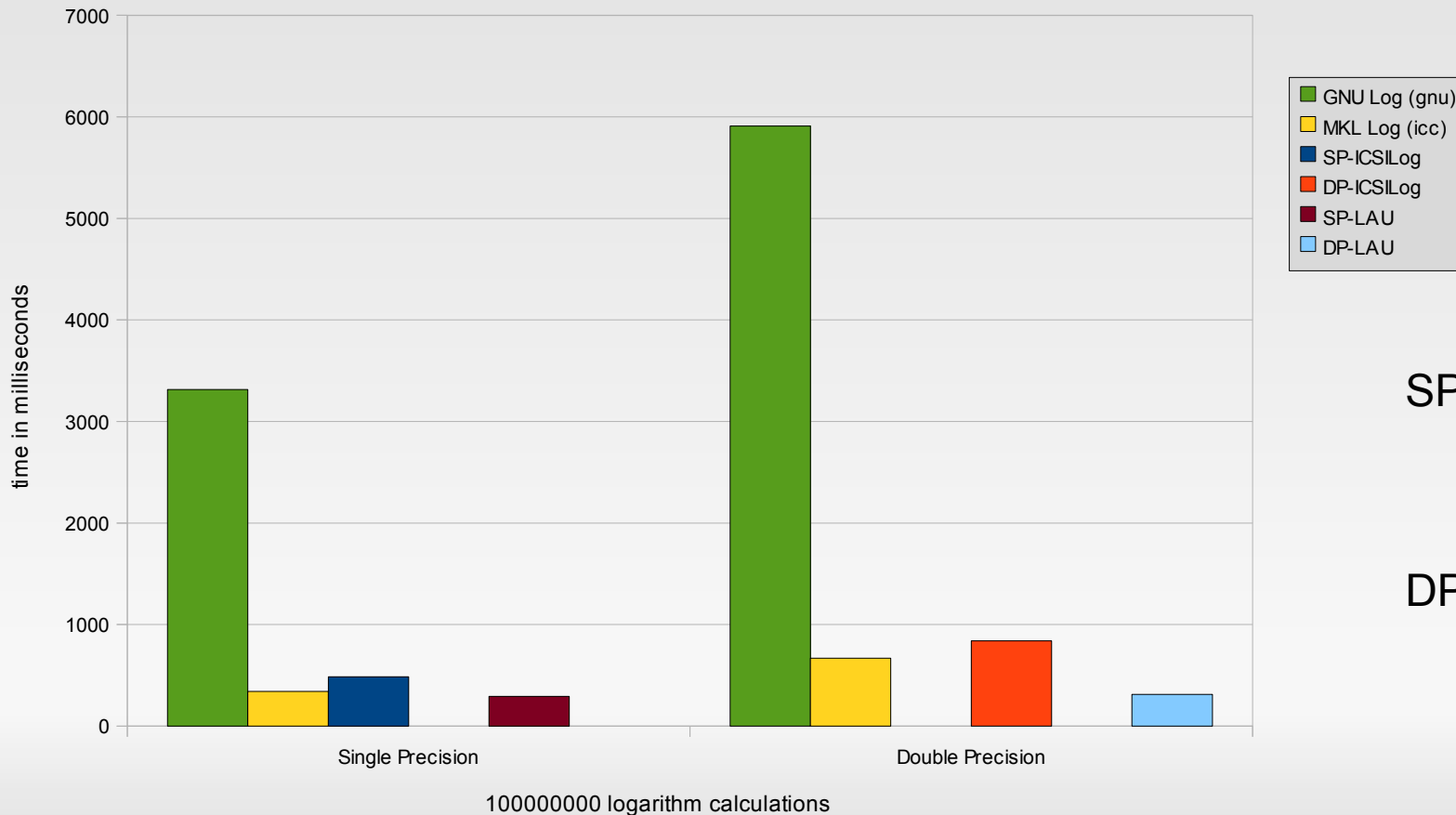


## Performance:

LAU vs SP/DP-ICSILog vs GNU Log vs MKL Log



Intel Core2 DUO  
T9600 @ 2.8GHz  
6MB L2 Cache



SP-LAU VS  
GNU-LOG : 11X  
MKL-LOG : 1.6X

DP-LAU VS  
GNU-LOG: 18X  
MKL-LOG: 2.5X

# CONCLUSION and FUTURE WORK

## AVAILABILITY

DP-ICSILog C Implementation and SP/DP LAU FPGA core for Virtex4 and Virtex5 FPGAs

**<http://www.krammer.in.tum.de/exelixis/nikos/ipcores.html>**

Or

OpenCores.org: Project name: fp\_log

**[http://www.opencores.org/project,fp\\_log](http://www.opencores.org/project,fp_log)**

# CONCLUSION and FUTURE WORK

## RELATED PROJECTS

Implementation of a UDP/IP core for Virtex 5 FPGAs (optimized for PC-FPGA communication)

**<http://www.krammer.in.tum.de/exelixis/nikos/ipcores.html>**

Or

OpenCores.org: Project name: udp\_ip\_\_core

**[http://www.opencores.org/project,udp\\_ip\\_\\_core](http://www.opencores.org/project,udp_ip__core)**

## FUTURE WORK

Implementation of a resource-efficient exponential function

Integration of the LOG and EXP cores into the general Phylogenetic Architecture